The Pareto Frontier for Random Mechanisms

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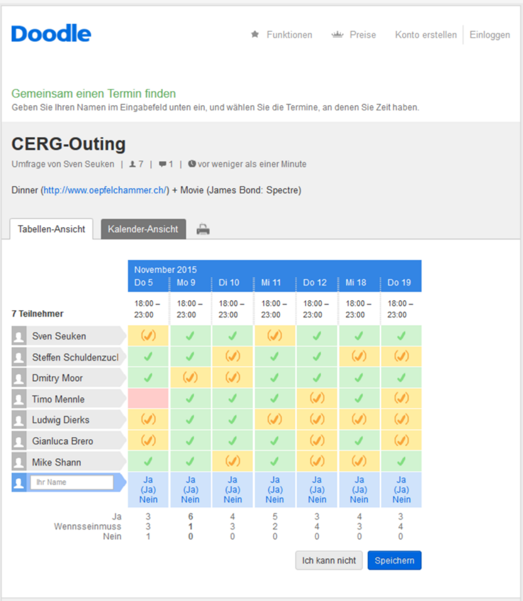
Meeting of COST Action on Computational Social Choice,
Istanbul Bilgi University, Turkey,
November 2, 2015

Motivating Example

- **Problem:** find date for CERG social evening
- **Setting:** 7 agents, 8 alternatives
- **Preferences:** agents classify alternatives into *good*, *acceptable*, or *unacceptable*
- **Desideratum ("Doodle"):**
  1. maximum participation
  2. *good* for many agents



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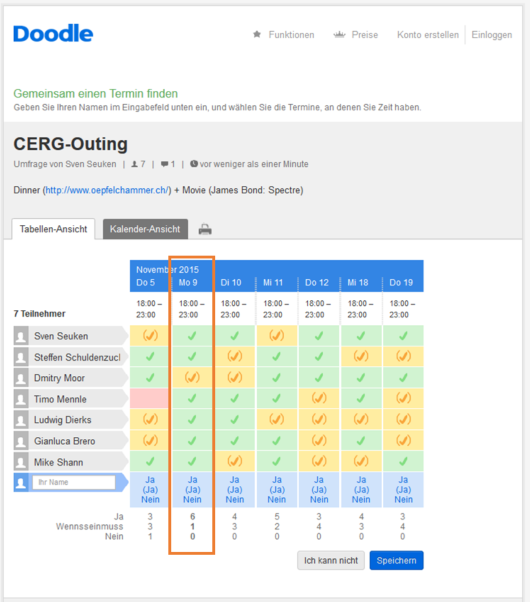

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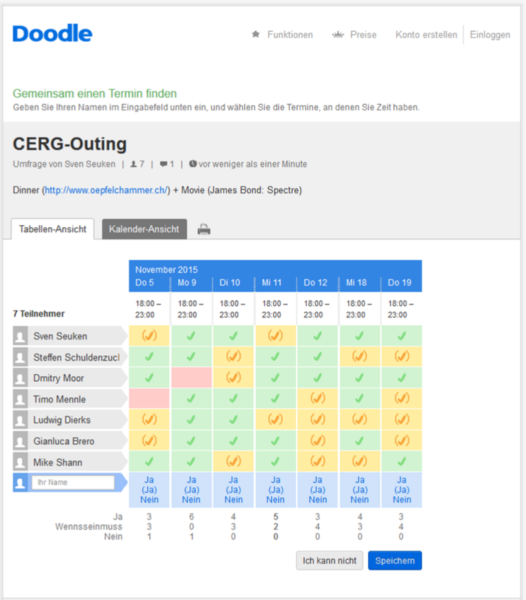

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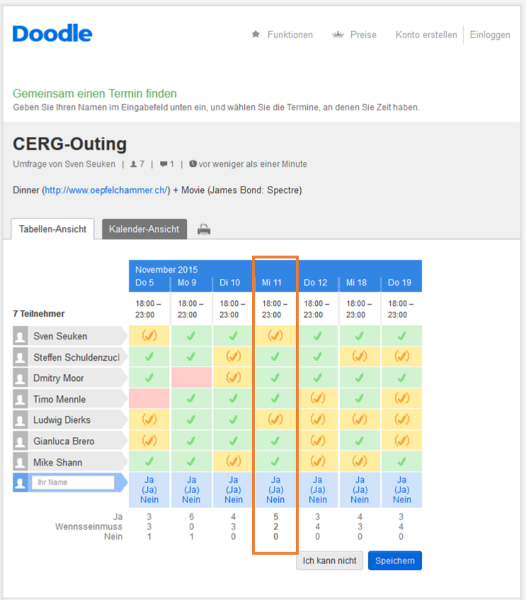

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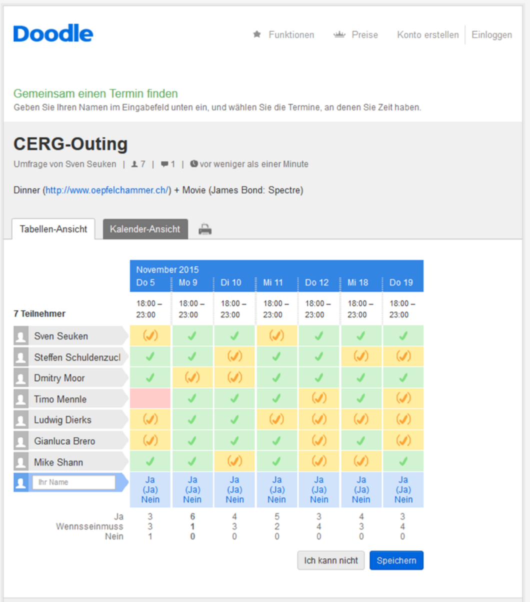

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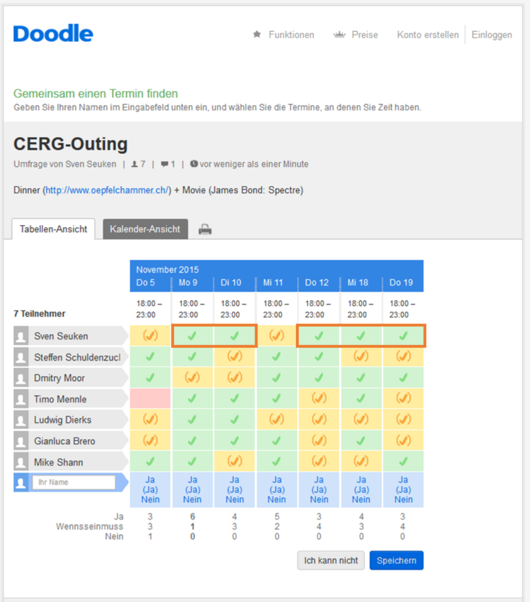

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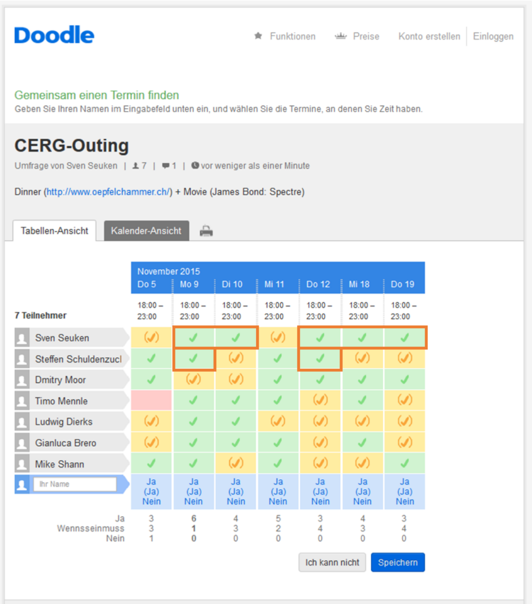

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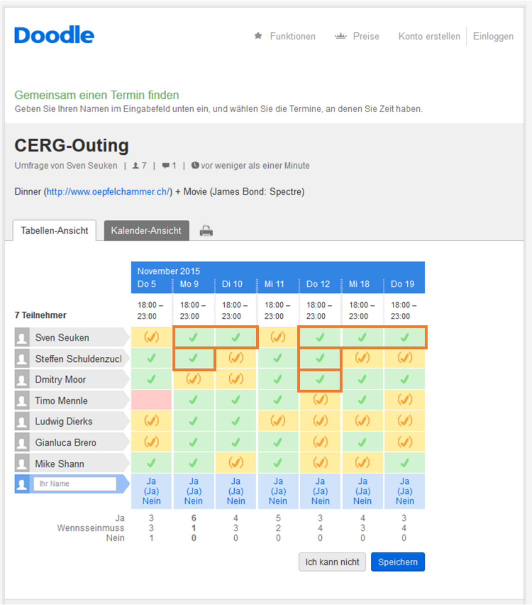

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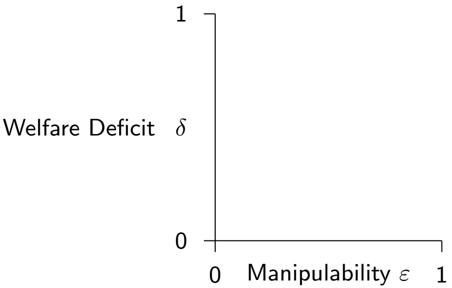

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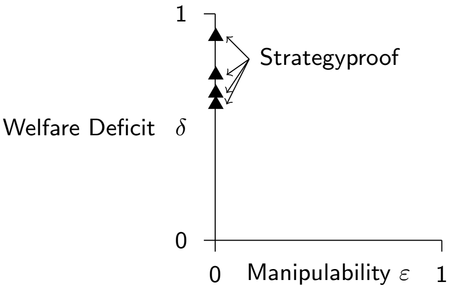

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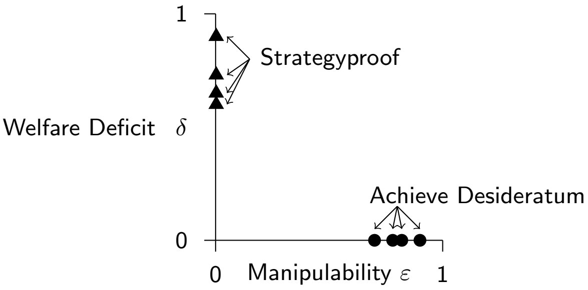

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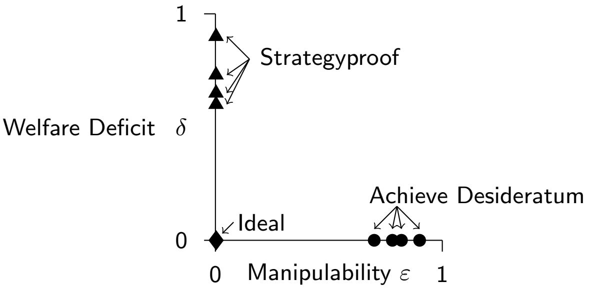

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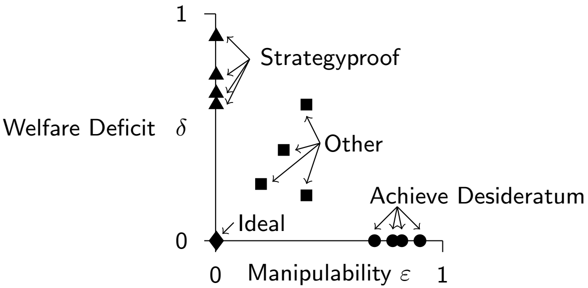

Main Result

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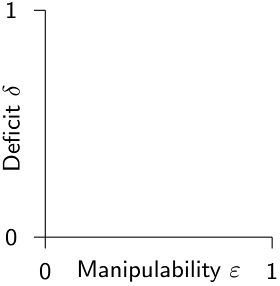

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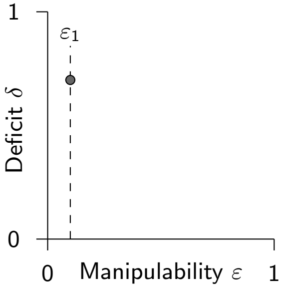

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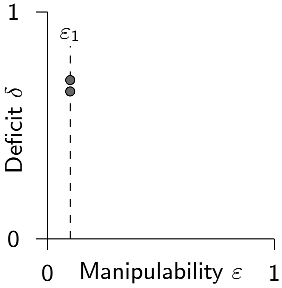

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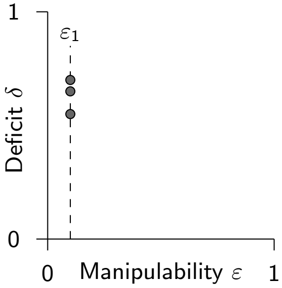

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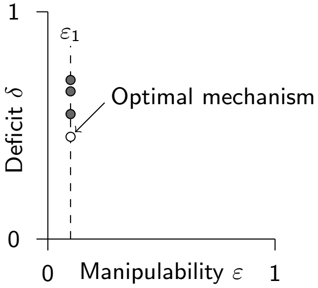

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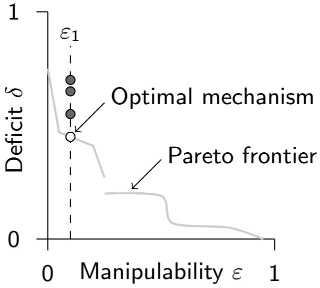

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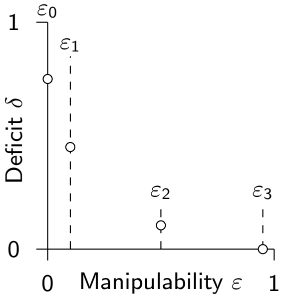

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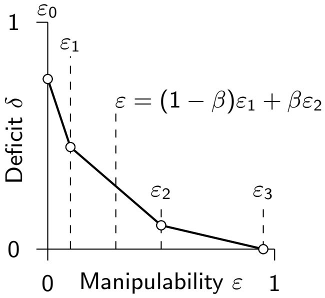

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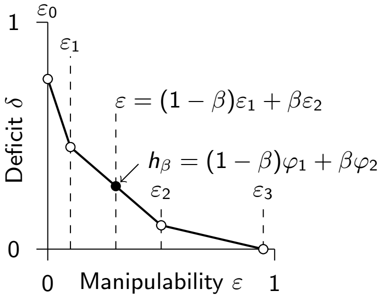

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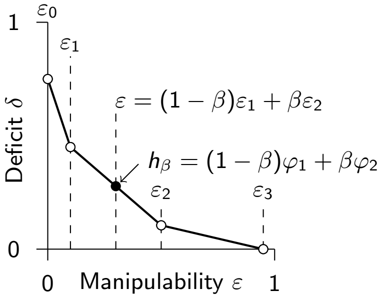

Agenda

1. Preliminaries
2. #1 Optimal Mechanisms
3. #2 Hybrid Mechanisms
4. Pareto Frontier
5. Generality and Conclusion

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Related Work

Domain restrictions:

- Single-peaked preferences (Moulin, 1980)
- Random assignment (Hylland and Zeckhauser, 1979; Abdulkadiroğlu and Sönmez, 1998; Bogomolnaia and Moulin, 2001)

This talk: full & restricted domains

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Trade-offs:

- Approximation of efficient mechanisms in large markets (Procaccia, 2010; Birrell and Pass, 2011)
- SD-efficiency versus (weak) SD-strategyproofness of random mechanisms (Aziz, Brandl and Brandt, 2014)

This talk: finite setting & exact trade-offs

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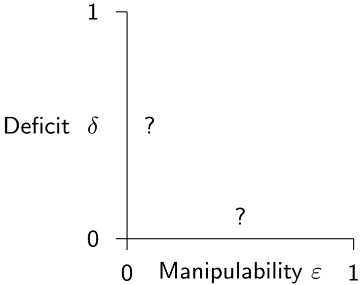
- *Random mechanism* $\varphi : \mathcal{P}^n \rightarrow \Delta(M)$
  $\varphi(\boldsymbol{P}) = (x_1, \ldots, x_m)$, where $\varphi_j(\boldsymbol{P}) = x_j$ probability for j

Measures



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- *Manipulability* of mechanism φ

$$\varepsilon(\varphi) = \min\{\varepsilon' \in [0,1] : \varphi \text{ is } \varepsilon'\text{-approximately strategyproof}\}$$

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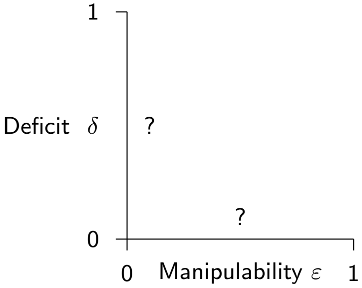

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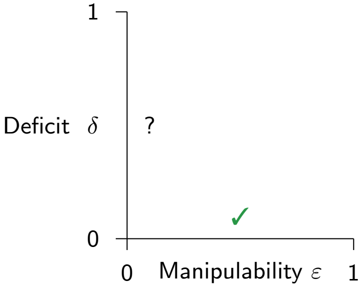

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- Welfare of lottery $\boldsymbol{x}$: $w(\boldsymbol{x}, \boldsymbol{P}) = \sum_{j \in M} w(j, \boldsymbol{P}) \cdot x_j$
  Interpretation: *expected score*, *probability of optimality*

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  - *Pareto optimality*: $w(j, \boldsymbol{P}) = \mathbb{1}_{\{j \text{ not Pareto dominated at } \boldsymbol{P}\}}$

    ...or *doodle:*

$$w(j, \boldsymbol{P}) = \begin{cases} \#\{i : j \text{ good for } i\}/n, & \text{if participation maximal at } j, \\ 0, & \text{else} \end{cases}$$

- Welfare of lottery $\boldsymbol{x}$: $w(\boldsymbol{x}, \boldsymbol{P}) = \sum_{j \in M} w(j, \boldsymbol{P}) \cdot x_j$
  Interpretation: *expected score*, *probability of optimality*

Measuring performance

- *Welfare deficit* of lottery $\boldsymbol{x}$ at $\boldsymbol{P}$

$$\delta(\boldsymbol{x}, \boldsymbol{P}) = \max_{j \in M} (w(j, \boldsymbol{P})) - w(\boldsymbol{x}, \boldsymbol{P})$$

Desideratum: Welfare & Deficit

Encoding desiderata

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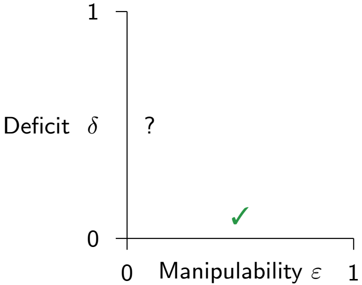
- *Welfare deficit* of mechanism φ

$$\delta(\varphi) = \max_{\boldsymbol{P} \in \mathcal{P}^n} \delta(\varphi(\boldsymbol{P}), \boldsymbol{P})$$

Measures



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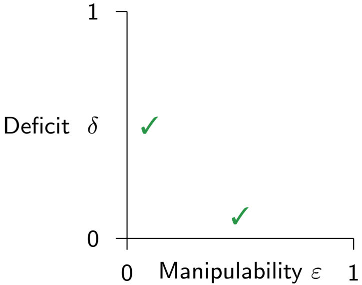

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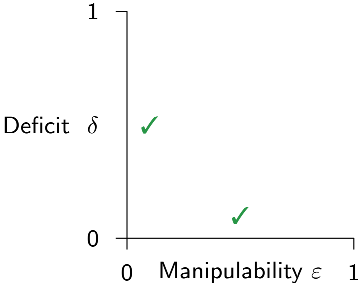

- Fix *problem* (N, M, δ)

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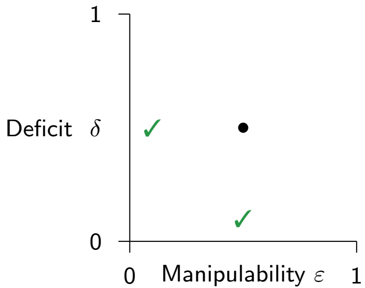
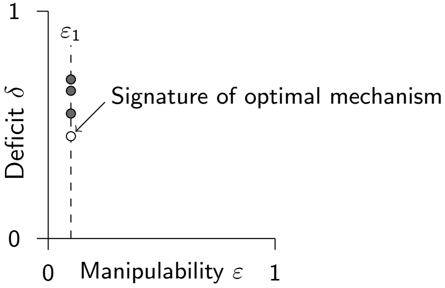


- Fix *problem* (N, M, δ)
- $(\varepsilon(\varphi), \delta(\varphi)) \in [0,1] \times [0,1]$ *signature*

Agenda

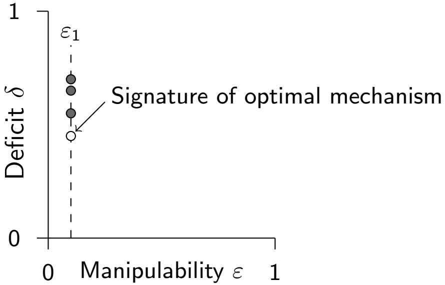
#1 Optimal Mechanisms



Definition (Optimal mechanisms)

$$\textsc{Opt}(\varepsilon) = \{\varphi : \varphi \text{ is } \varepsilon\text{-ASP}, \delta(\varphi) = \min\{\delta(\varphi') : \varphi' \text{ is } \varepsilon\text{-ASP}\}\}$$

#1 Optimal Mechanisms



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- $\textsc{Opt}(\varepsilon) \neq \emptyset$

#1 Optimal Mechanisms

$\textsc{Opt}(\varepsilon)$ solution to optimization problem:

$$\begin{aligned} \text{minimize} \quad & \delta(\varphi) && \text{(minimize deficit)} \\ \text{subject to} \quad & \varphi \text{ random mechanism} \\ & \varphi\ \varepsilon\text{-approx. SP} \end{aligned}$$

#1 Optimal Mechanisms

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Theorem (Finite Equivalent Set of Linear Constraints)

φ *ε-approximately strategyproof* if and only if *for all profiles $\boldsymbol{P} \in \mathcal{P}^n$, agents $i \in N$, misreports $P_i' \in \mathcal{P}$, indices $k \in \{1, \ldots, m\}$:*

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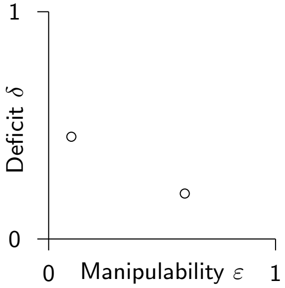
Agenda

#2 Hybrid Mechanisms



Idea: "mix" mechanisms for intermediate signatures

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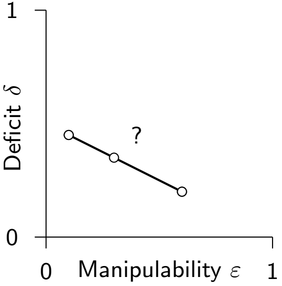

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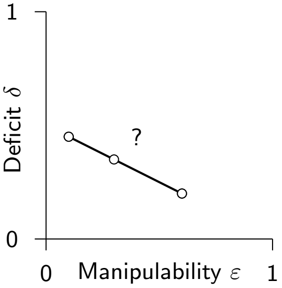

Idea: "mix" mechanisms for intermediate signatures

Definition (β-hybrid)

$h_\beta = (1-\beta)\varphi + \beta\psi, \beta \in [0,1]$

#2 Hybrid Mechanisms

Theorem (Guarantees for Hybrids)

For any mechanisms φ, ψ *and* $\beta \in [0, 1]$,

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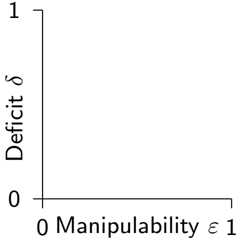

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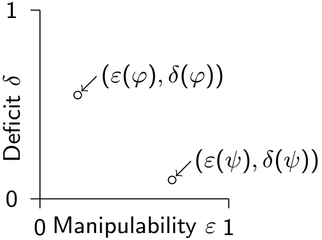

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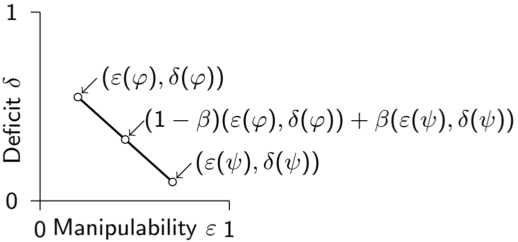

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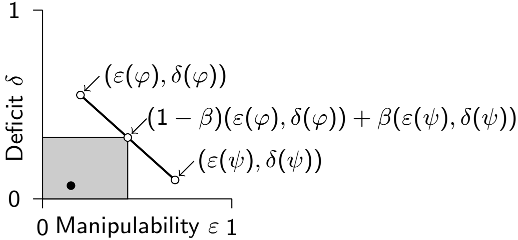

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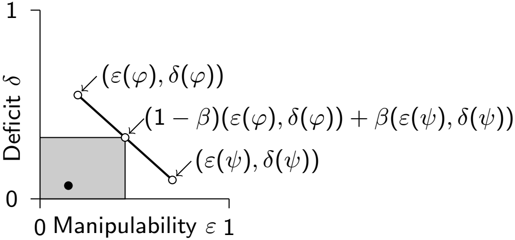


→ Anonymity and neutrality are "free"

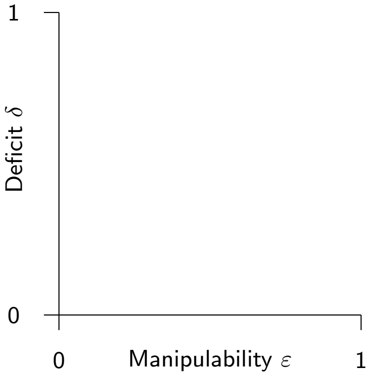
Agenda

1. Preliminaries
2. #1 Optimal Mechanisms
3. #2 Hybrid Mechanisms
4. Pareto Frontier
5. Generality and Conclusion

Pareto Frontier - Characterization Result

Result (informal):

- *Pareto frontier* set of optimal mechanisms



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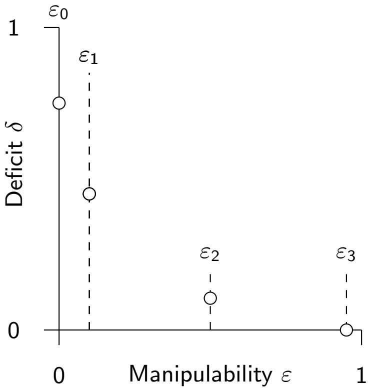

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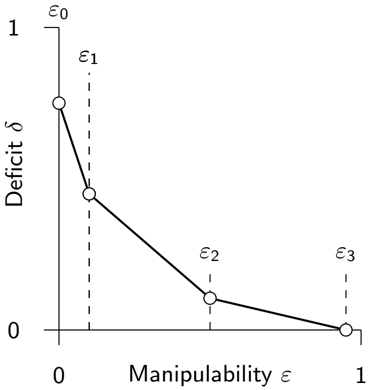

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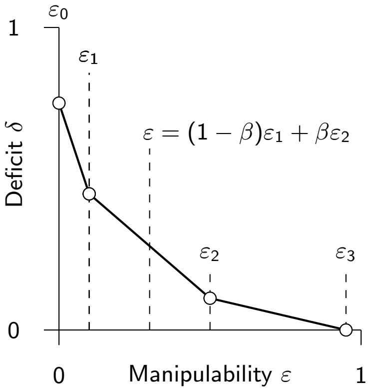

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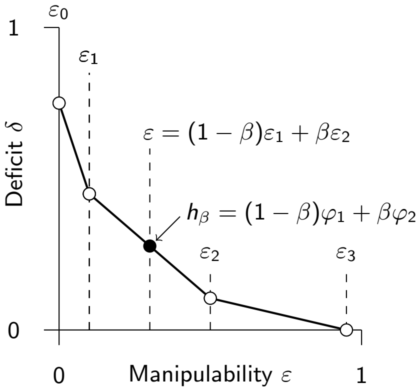

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Theorem (Characterization of $\textsc{Pf}$)

Given a problem (N, M, δ), there exist finitely many supporting manipulability bounds

$$0 = \varepsilon_0 < \ldots < \varepsilon_K = \bar{\varepsilon},$$

such that for any $[\varepsilon_{k-1}, \varepsilon_k]$ and $\varepsilon = (1-\beta)\varepsilon_{k-1} + \beta\varepsilon_k$:

$$\begin{aligned} \textsc{Opt}(\varepsilon) &= (1-\beta)\textsc{Opt}(\varepsilon_{k-1}) + \beta\textsc{Opt}(\varepsilon_k), \\ \delta(\varepsilon) &= (1-\beta)\delta(\varepsilon_{k-1}) + \beta\delta(\varepsilon_k). \end{aligned}$$

Pareto Frontier - Trade-offs

Properties of signature plot $\varepsilon \mapsto \delta(\varepsilon)$

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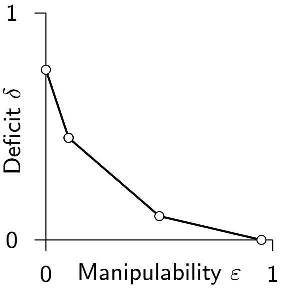
- monotonic & decreasing $\rightarrow$ trade-offs



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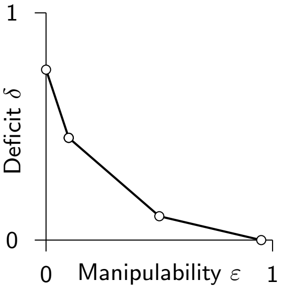

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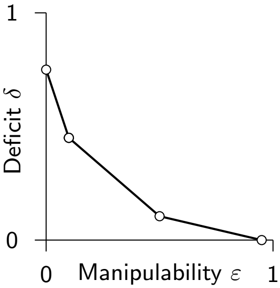
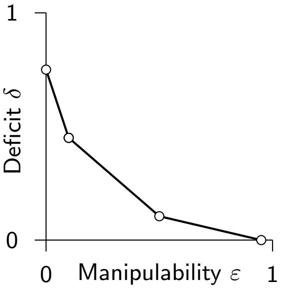


Pareto Frontier - Trade-offs

Properties of signature plot $\varepsilon \mapsto \delta(\varepsilon)$

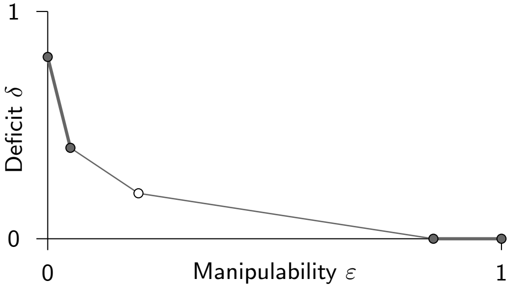
- monotonic & decreasing $\rightarrow$ trade-offs
- convex $\rightarrow$ decreasing marginal reduction of deficit
- continuous $\rightarrow$ smooth change
- piecewise linear $\rightarrow$ $\textsc{Pf}$ computable



Pareto Frontier - Computation

Algorithm compute $\text{OPT}(\varepsilon_k)$ for all $k \in \{0, \ldots, K\}$:

- Interpolate $\rightarrow$ potential supporting manipulability bound ε
- Compute $\delta(\varepsilon) \rightarrow$ verify/discard ε
- Repeat ...



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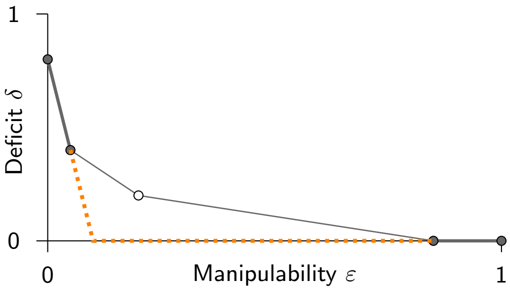

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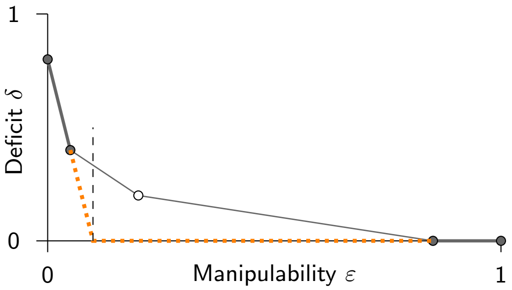

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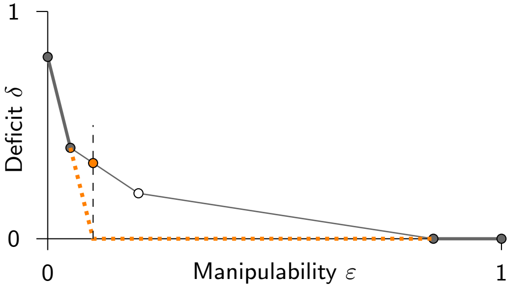

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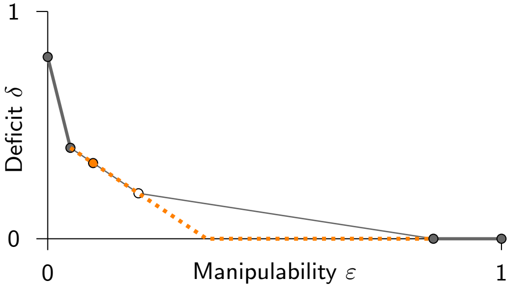

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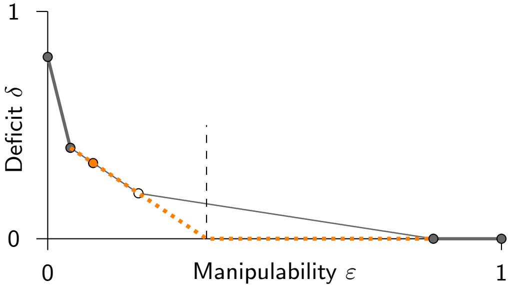

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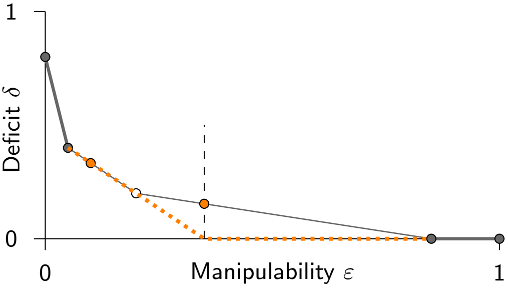

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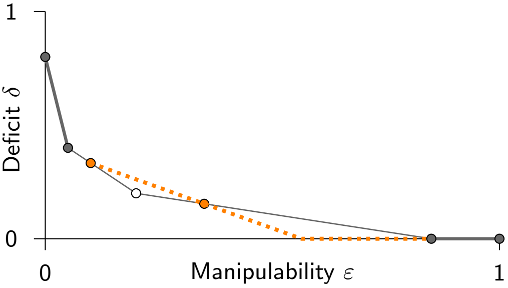

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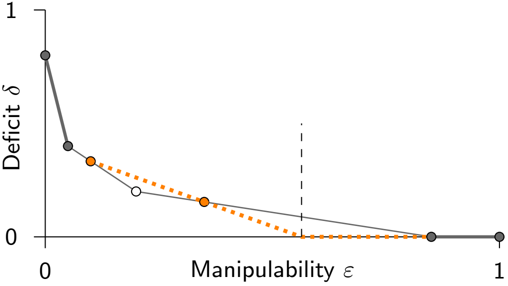

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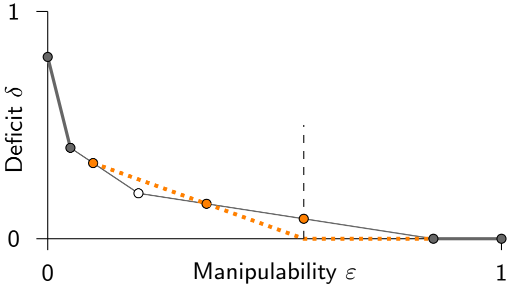

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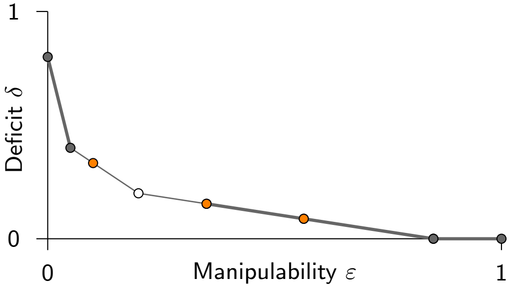

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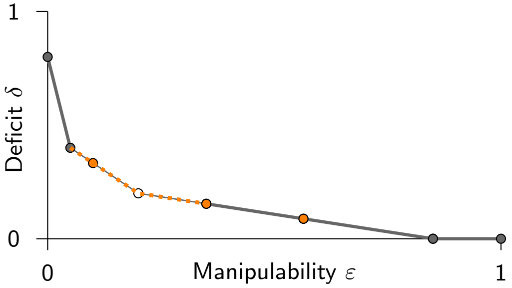

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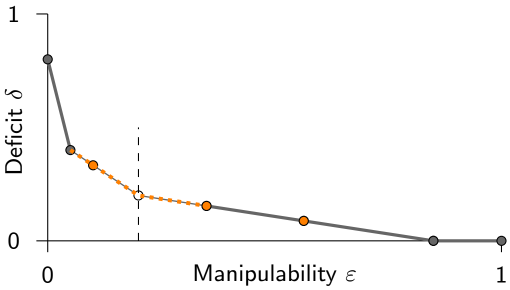

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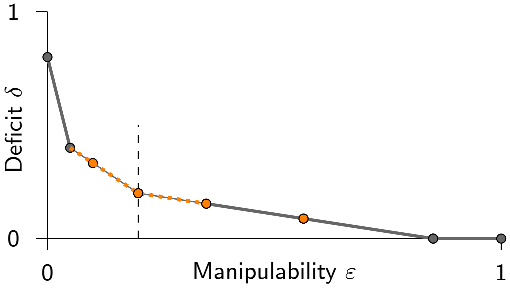

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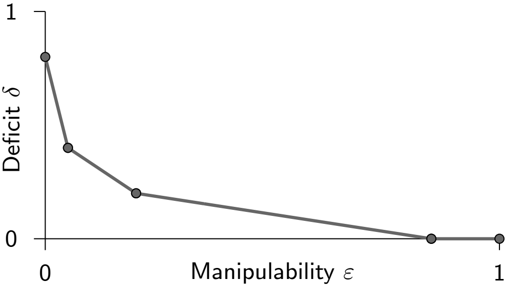

Pareto Frontier - Computation

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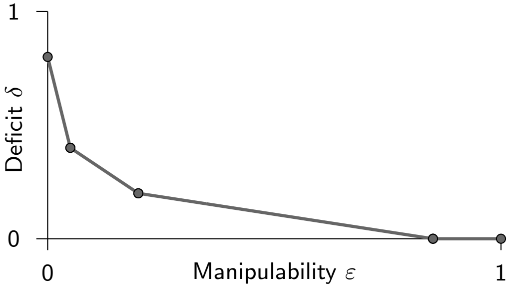

Runtime: at most $4K + \log_2(1/\varepsilon_1) - 1$ executions of LP

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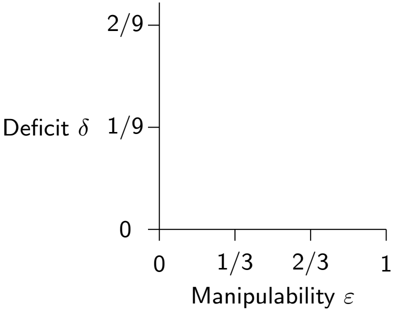
Problem: 3 agents, 3 alternatives, only strict preferences, welfare function: positional scoring $v = (1, 0, 0)$ (Plurality)



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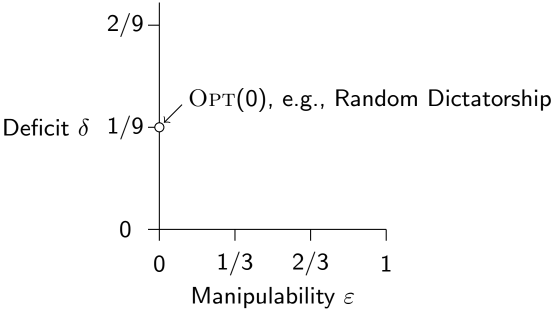

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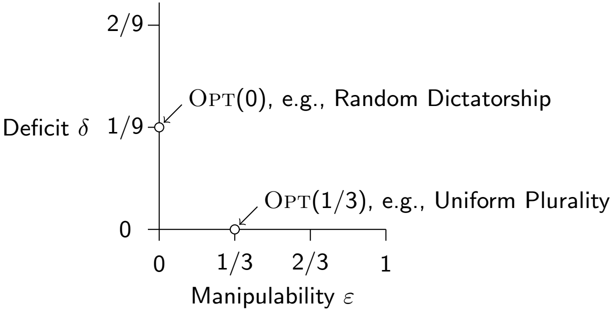

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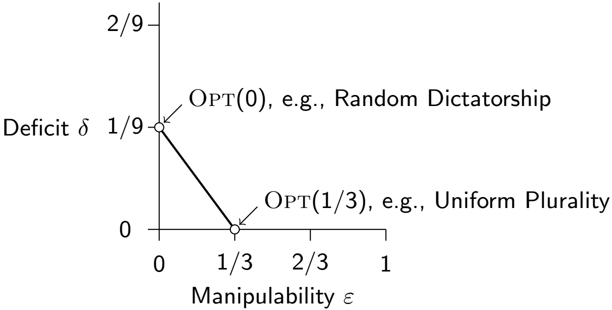

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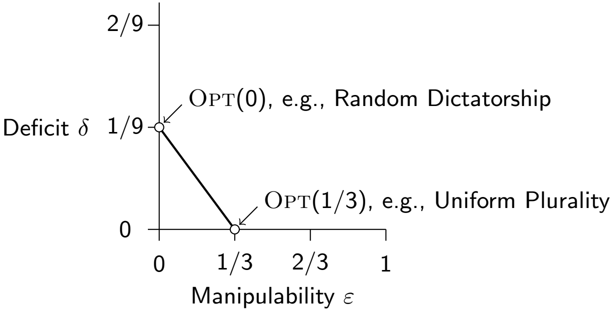

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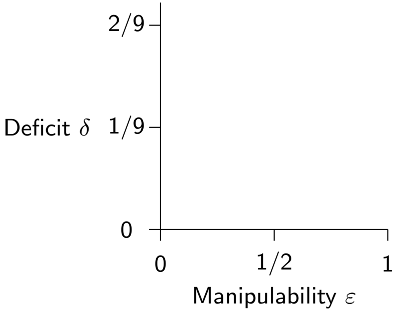

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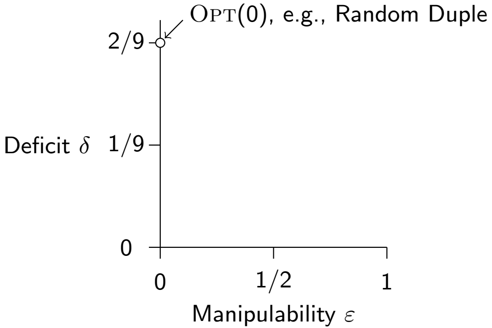

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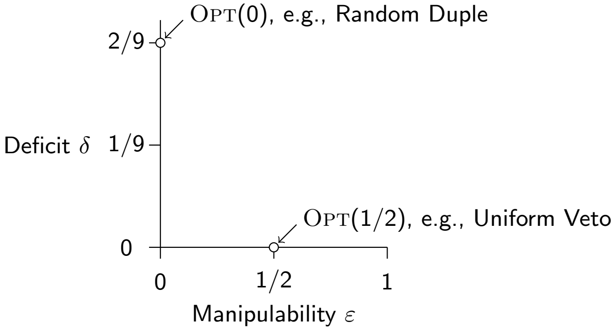

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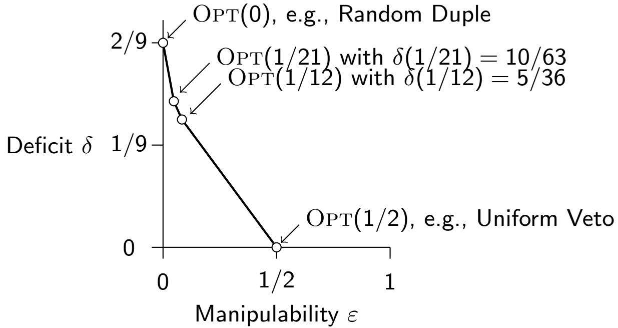

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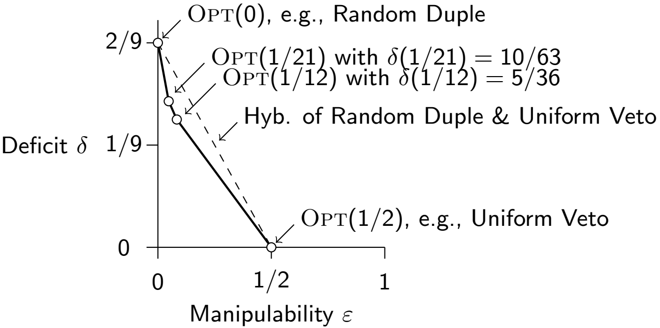

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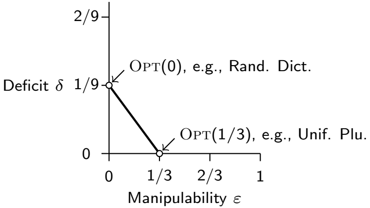

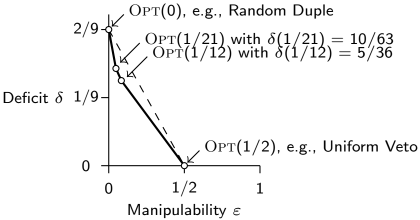

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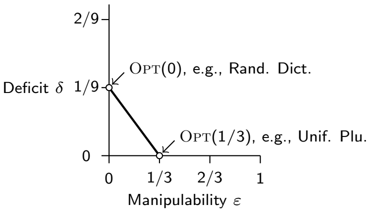

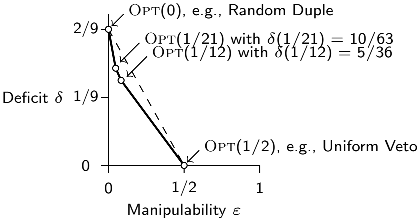

Thank you!

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