Uncertainty and Bounded-Rationality in Voting

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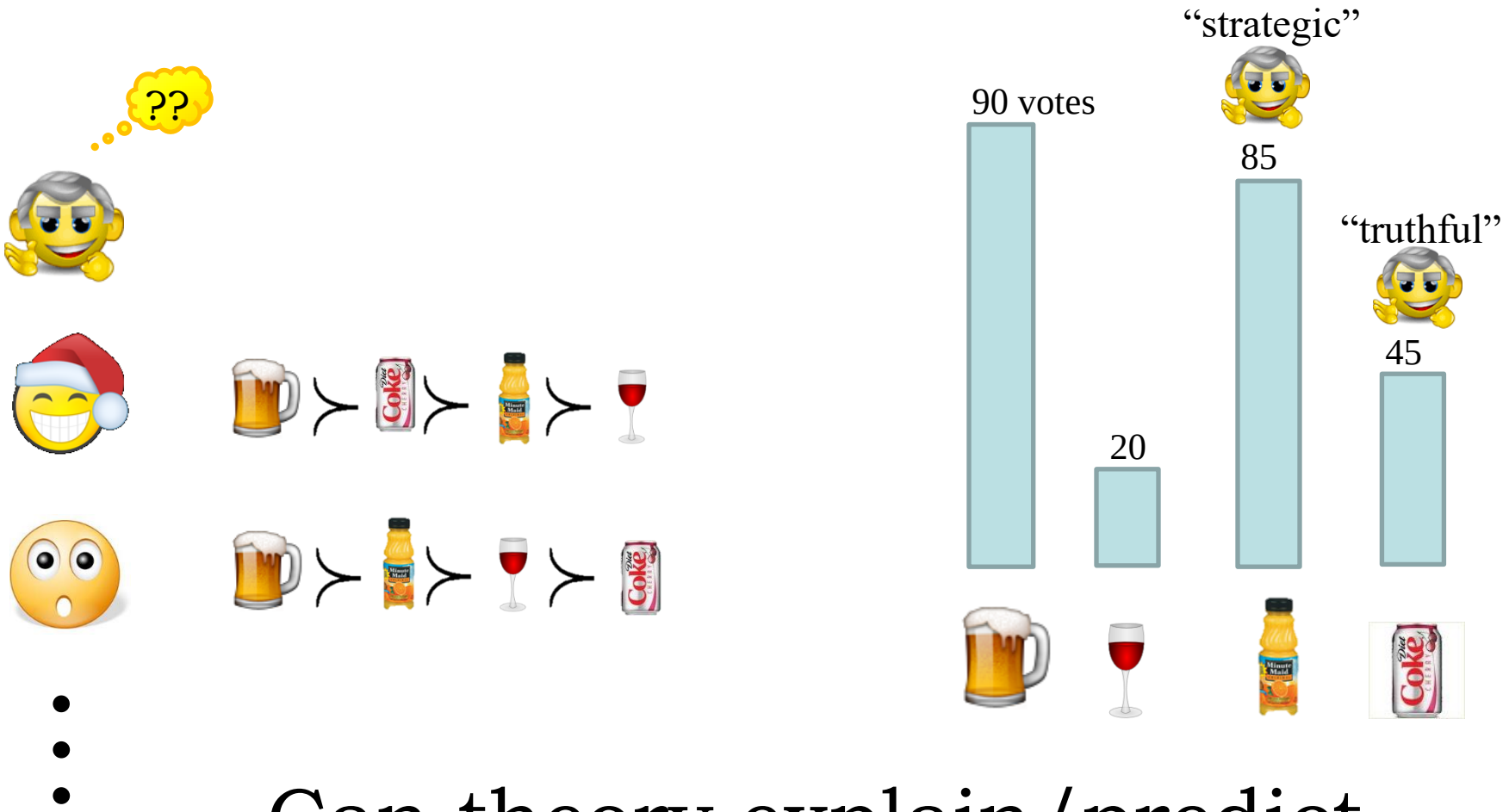
Based on joint work(s) with Omer Lev, David Parkes, Jeff Rosenschein, and James Zou

Plurality voting - example



Can theory explain/predict voting behavior?

(arguable) Desiderata for voting models

	"Leader rule" [Laslier'09]	Expected utility [MW'93,MP'02,…]
• Theoretic criteria (Rationality, equilibrium)	V	V
• Behavioral criteria (voters' beliefs and capabilities)	Bounded rationality V	X
• Scientific criteria: (Robustness, Simplicity, consistent with data, Discriminative power)	V	

Our contribution

Formal and empirical results

Behavioral model
(for limited capabilities)

Epistemic model
(for limited information)

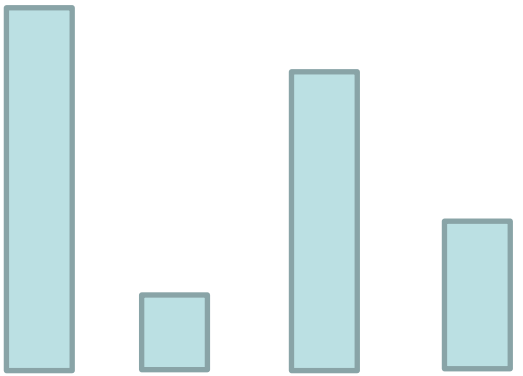
Epistemic model

Prospective scores $\boldsymbol{s}$

- E.g. from a poll
- "world state"

Uncertainty level $r_i \geq 0$

$\boldsymbol{s} = (90,20,85,45)$
$\epsilon\ \mathbb{R}^{|\mathcal{C}|}$



$\mathcal{C} = \{$ 🍺 🍷 🧃 🥤 $\}$

[Simon'57]:

"...the state of information may as well be regarded as a ***characteristic of the decision-maker*** *as a characteristic of his environment"*

Epistemic model

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- E.g. from a poll
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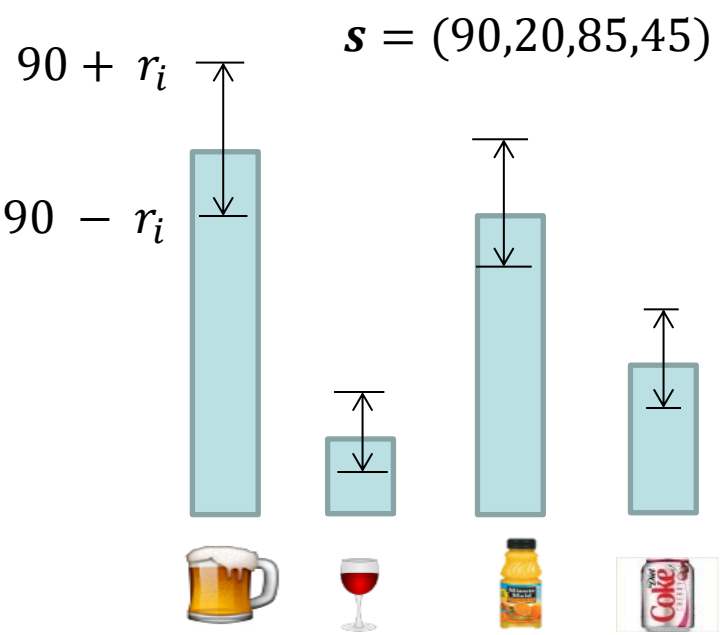

Voter i considers as "possible" all states close enough to $\boldsymbol{s}$. $S(\boldsymbol{s}, r_i) = \{\boldsymbol{s}' : \|\boldsymbol{s}' - \boldsymbol{s}\| \leq r_i\}$

– Example I: "*additive uncertainty*"

Epistemic model

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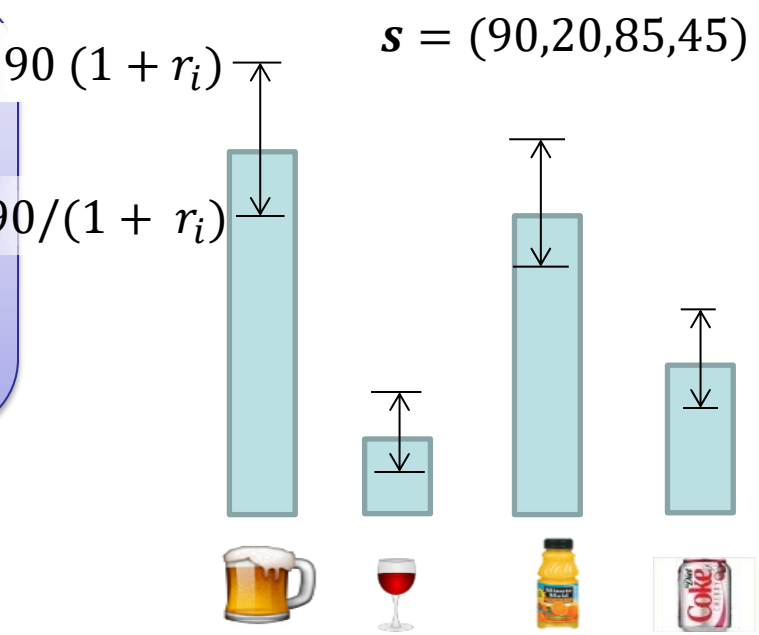

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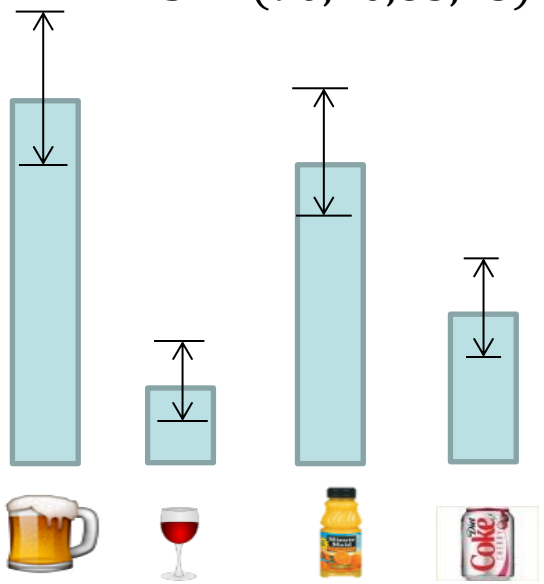
– Example I: "*additive uncertainty*"

– Example II: "*multiplicative uncertainty*"

Behavioral model

Rational agents avoid dominated strategies!

$\boldsymbol{s} = (90,20,85,45)$



$\boldsymbol{s} \in \mathbb{R}^{|\mathcal{C}|}$: state (scores)

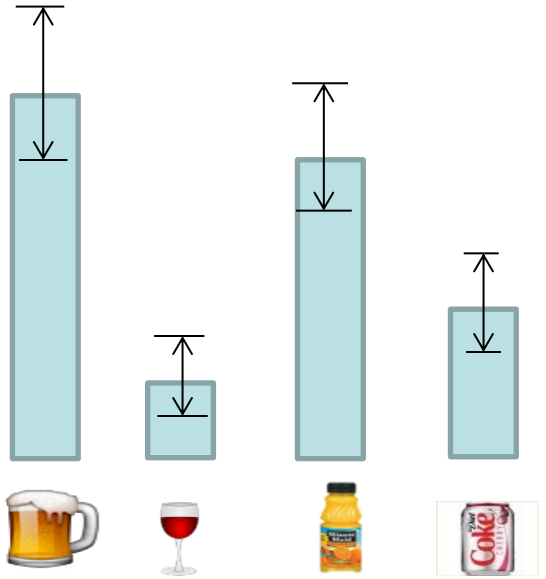
$S = S(\boldsymbol{s}, r_i)$: possible states

Def. I (*Local dominance*): A candidate c' *S-dominates* candidate c if it is always weakly better for i to vote for c'.

in every state $\boldsymbol{s}' \in S$

Behavioral model

Rational agents avoid dominated strategies!



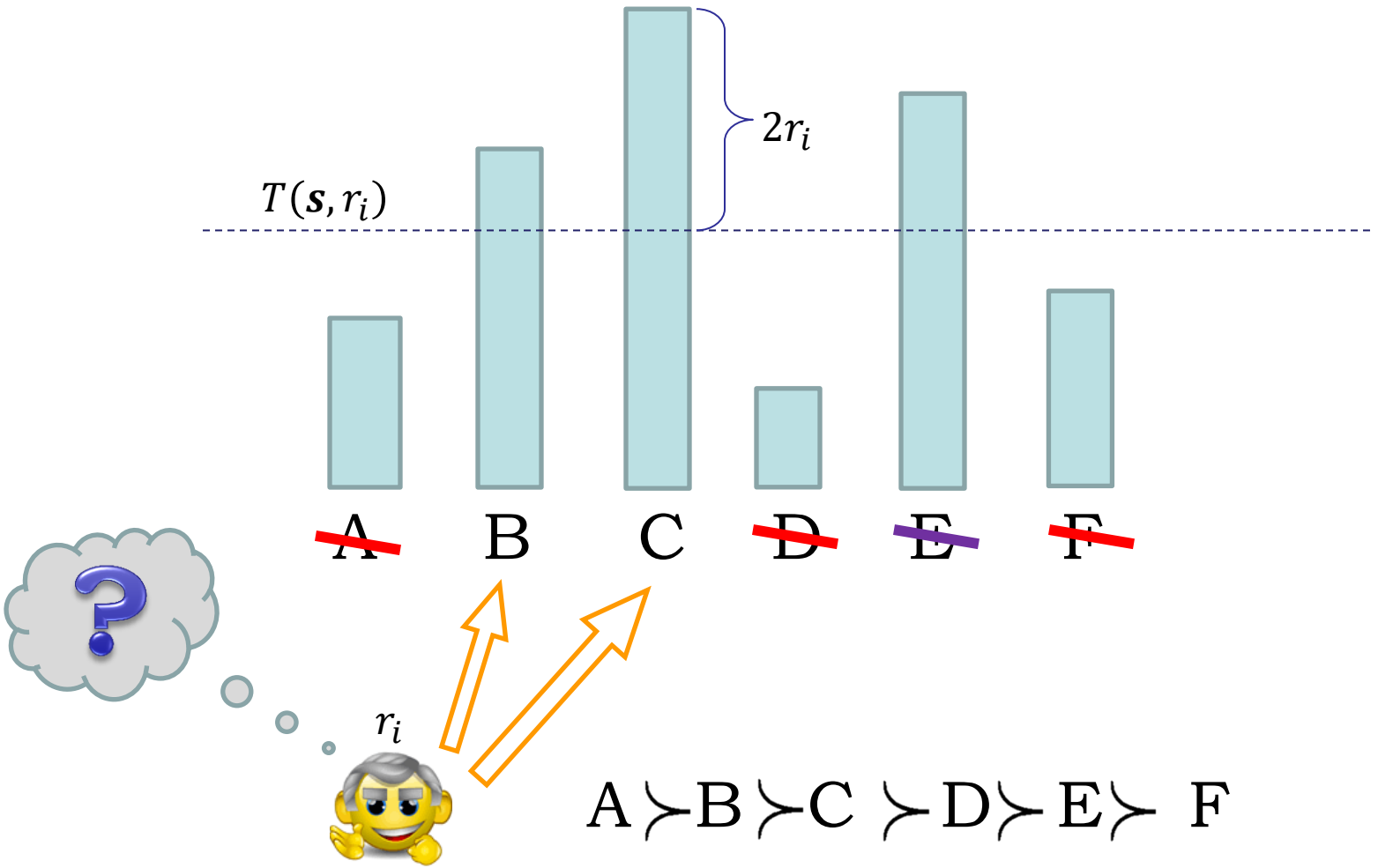
<u>One-shot voting</u>:
Vote for a candidate that is not locally-dominated

<u>Iterative setting</u>: As long as your vote is locally dominated, switch to a candidate that dominates it. **Otherwise – stay.** *Local dominance move*

Strategic voting (one shot)

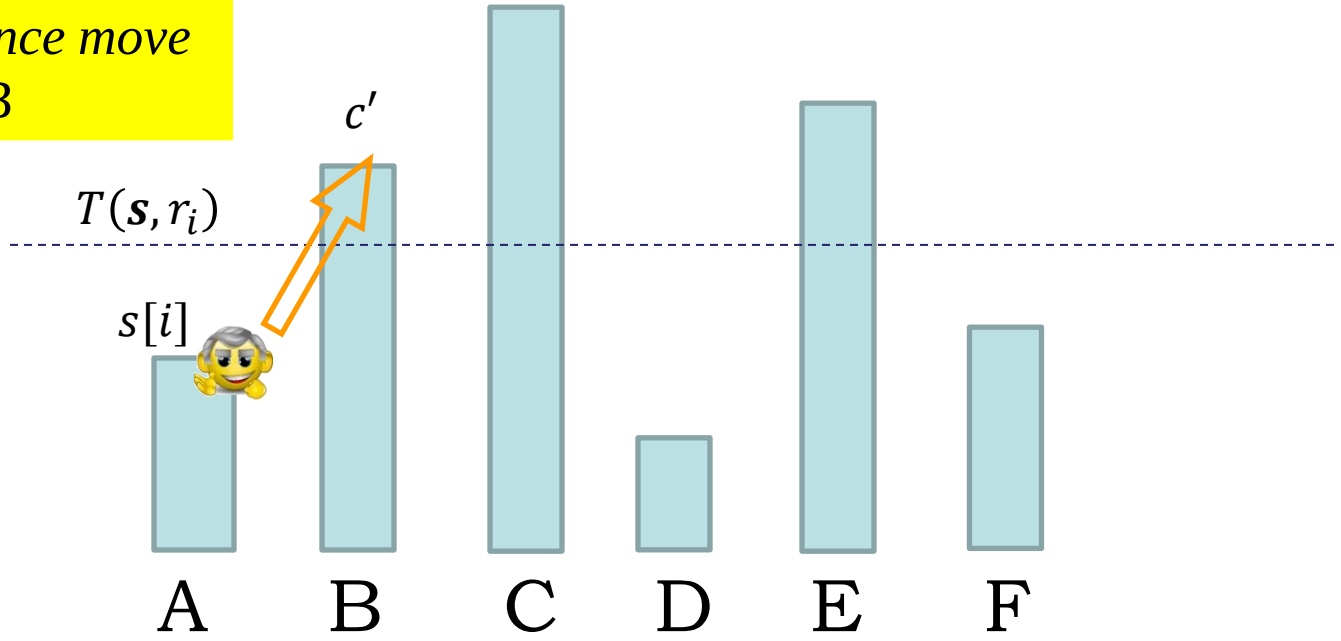
Lemma: All dominance relations in state $\boldsymbol{s}$ are characterized by a single threshold $T(\boldsymbol{s}, r_i)$: (depends on winner's score)

c **is dominated** iff **below the threshold** *or* **least preferred**.*



Strategic voting (iterative)

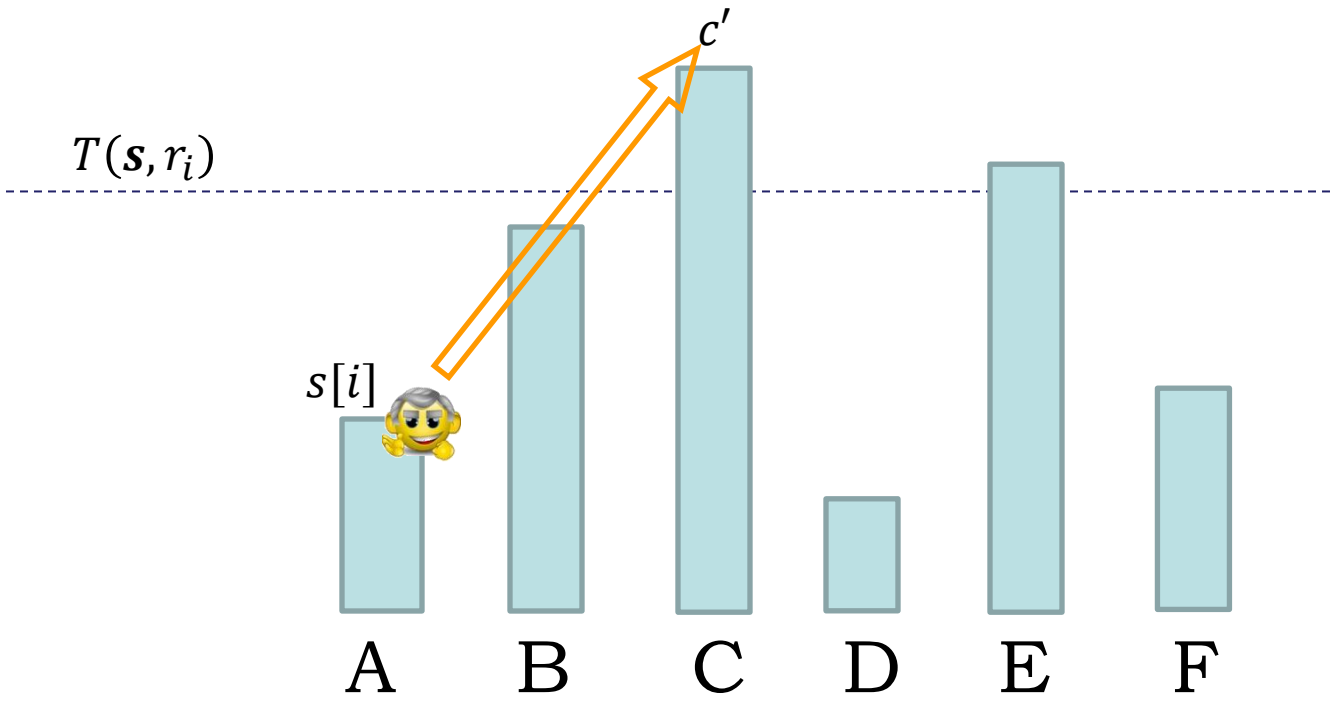
Local dominance move
A → B



$s[i] \in \mathcal{C}$: the vote of voter i in state $\boldsymbol{s}$

$A \succ B \succ C \succ D \succ E \succ F$

Strategic voting (iterative)



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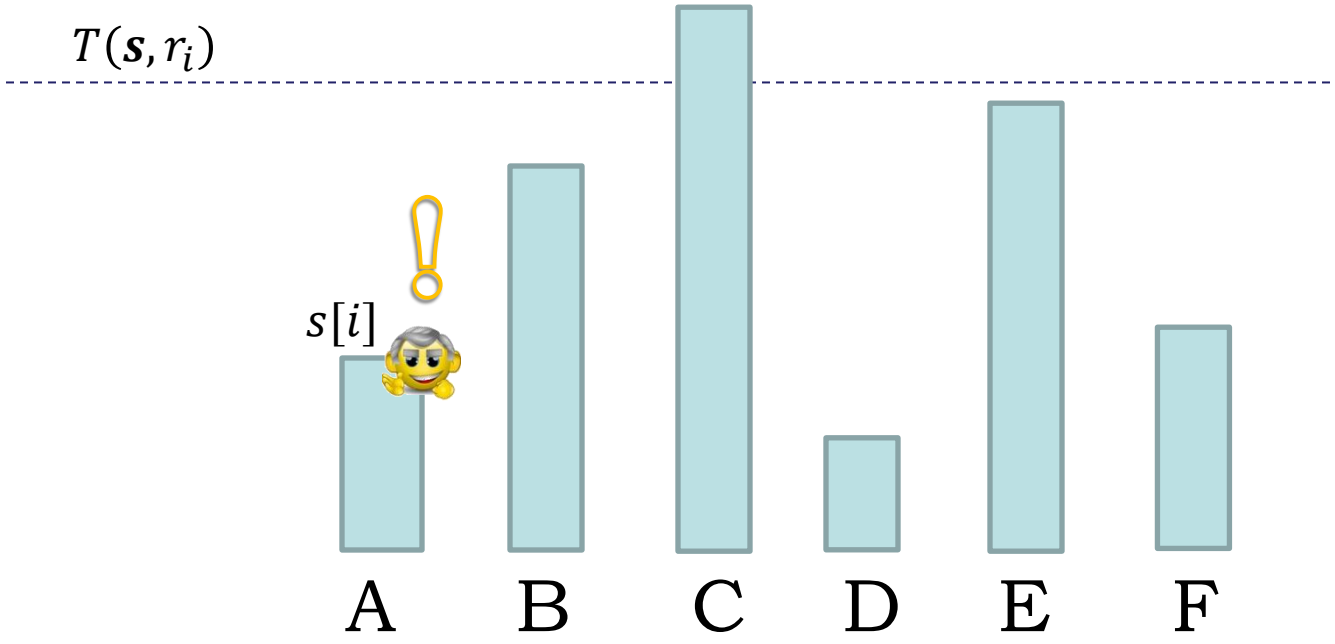

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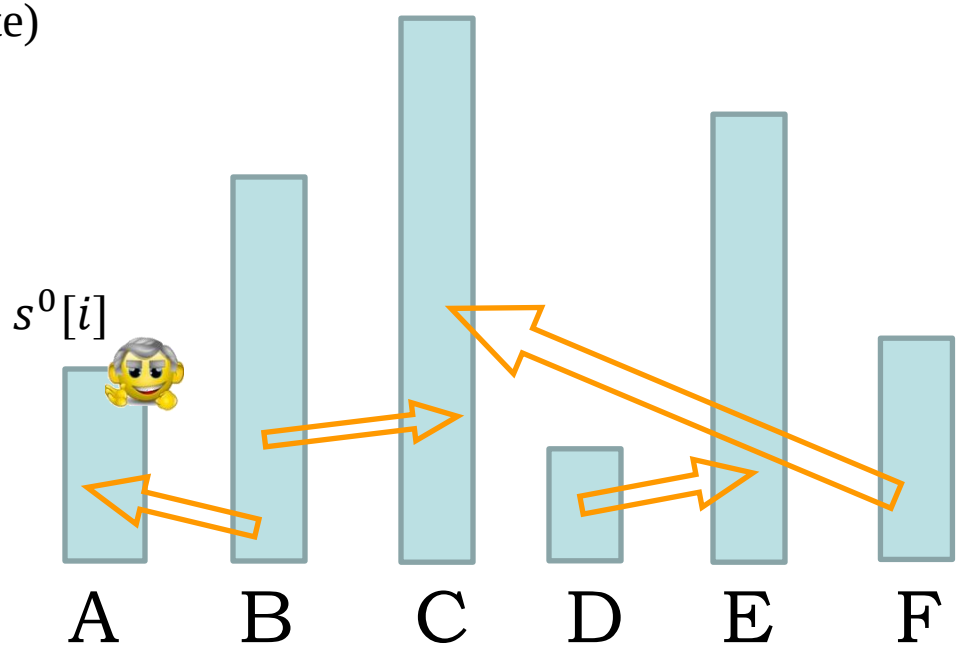
Strategic voting (iterative)

$\boldsymbol{s}^0$ (initial state)



$s^t[i] \in \mathcal{C}$: the vote of voter i in state $\boldsymbol{s}^t$

$A \succ B \succ C \succ D \succ E \succ F$

Strategic voting (iterative)

Def. II (*voting equilibrium*):
A state $\boldsymbol{s}$ where for every voter i, the candidate $s[i]$ is not $S(\boldsymbol{s}, r_i)$-dominated.

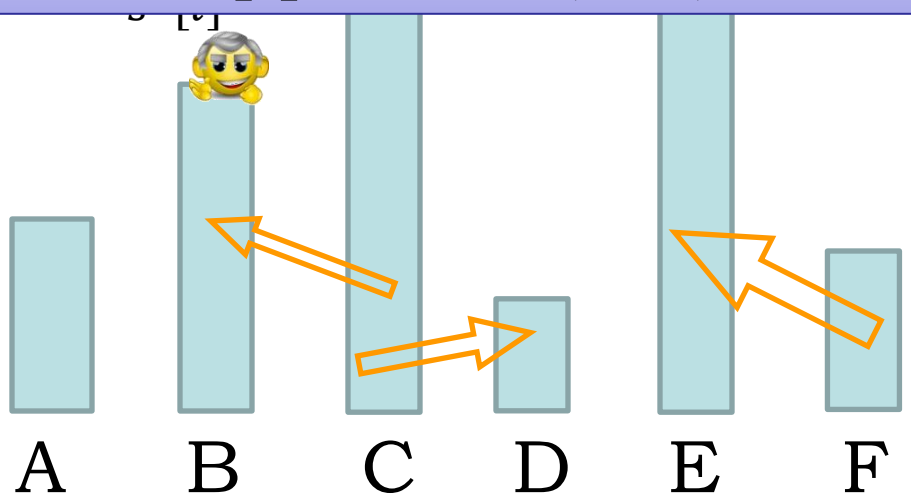
$\boldsymbol{s}^0$ (initial

$\boldsymbol{s}^1$

$\boldsymbol{s}^2$

$\boldsymbol{s}^3$

$\vdots$



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Def. II (*voting equilibrium*):
A state $\boldsymbol{s}$ where for every voter i, the candidate $s[i]$ is not $S(\boldsymbol{s}, r_i)$-dominated.

- Existence?

Independent of voting order

- Convergence?

Prop. *[M., Polukarov, Rosenschein, Jennings, AAAI'10]: "**best-response** in voting converges to a **Nash equilibrium**."*

- Properties?

Results

Main Theorem *[M. AAAI'15]*:

Any sequence $\boldsymbol{s}^0 \rightarrow \boldsymbol{s}^1 \rightarrow \boldsymbol{s}^2 \rightarrow \cdots$ of Local-dominance moves is acyclic (must converge).

In particular, a voting equilibrium always exists.

- From any initial state $\boldsymbol{s}^0$
- Uncertainty levels r_i may be diverse
- Arbitrary order of players
- For a *nonatomic model*: Also holds under simultaneous moves

Results

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Prop. *[M., Polukarov, Rosenschein, Jennings, AAAI'10]: "best-response in voting converges to a Nash equilibrium."*

Follows as a special case!

Proof sketch: $r_i = 0$ for all i $\Rightarrow$ $S(\boldsymbol{s}, r_i) = \{\boldsymbol{s}\}$

$\Rightarrow$ Local-dominance $\equiv$ Best response

$\Rightarrow$ Voting equilibrium $\equiv$ Nash equilibrium ■

Results

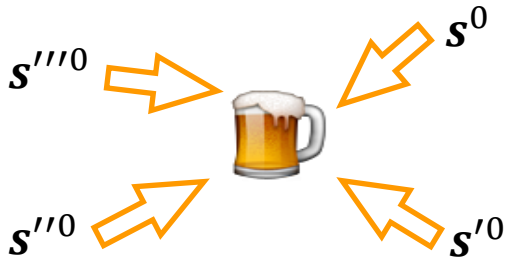
Def. II (*voting equilibrium*):
A state $\boldsymbol{s}$ where for every voter i, the candidate $s[i]$ is not $S(\boldsymbol{s}, r_i)$-dominated.

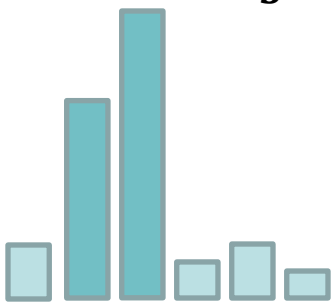
- Existence? ✓
- Convergence? ✓
- Properties?

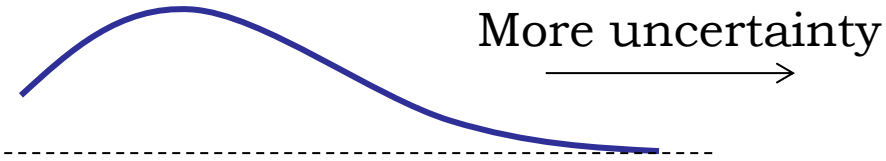
Extensive computer simulations:
>100 distributions of preferences
>10K profiles in total
>1M simulations

Results (computer simulations)

- Decisiveness
- Duverger Law
- Participation
- Welfare







[M., Lev, Rosenschein, EC'14]

Desiderata for voting models

Local-Dominance

- Theoretic criteria

  (Rationality, equilibrium) V

- Behavioral criteria

  (voters' beliefs and capabilities) V

- Scientific criteria:

  (Robustness, Simplicity, consistent with data, Discriminative power) V

Related work

- Voting experiments
  - VoteLib.org [Tal, M., Gal '15]

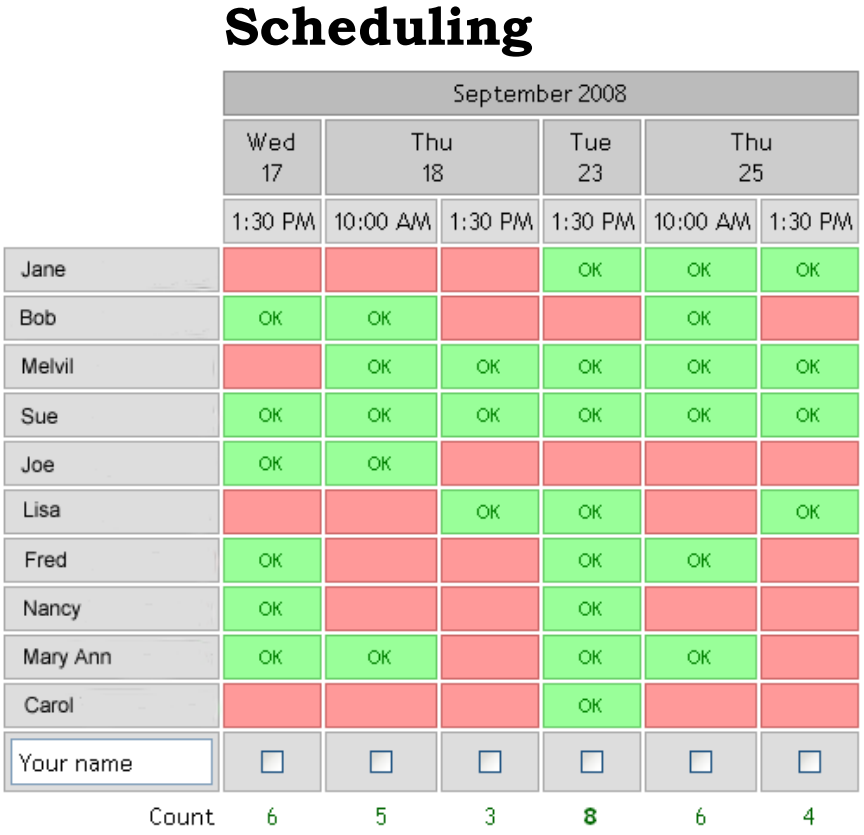
- Voting under strict uncertainty:
  - [Conitzer, Walsh, Xia '11] (dominance with information sets)
  - [Reijngoud, Endriss '12] (∏-manipulation)
  - [van Ditmarsch, Lang, Saffidine '13] (*de re* knowledge)

- Regret minimization [M.'15]
- Lazy/truth-biased voters […]
- Coordination in polls [Reyhani, Wilson, Khazaei '12]

What next?

- Doodle scheduling
- Uncertainty and Modal Logic
- Proof sketch for Plurality convergence

Doodle Scheduling



Scheduling

	September 2008					
	Wed 17	Thu 18		Tue 23	Thu 25	
	1:30 PM	10:00 AM	1:30 PM	1:30 PM	10:00 AM	1:30 PM
Jane				OK	OK	OK
Bob	OK	OK			OK	
Melvil		OK	OK	OK	OK	OK
Sue	OK	OK	OK	OK	OK	OK
Joe	OK	OK				
Lisa			OK	OK		OK
Fred	OK			OK	OK	
Nancy	OK			OK		
Mary Ann	OK	OK		OK	OK	
Carol				OK		
Your name	☐	☐	☐	☐	☐	☐
Count	6	5	3	8	6	4

Questions:

- Do people strategize when seeing previous responses?
- How?

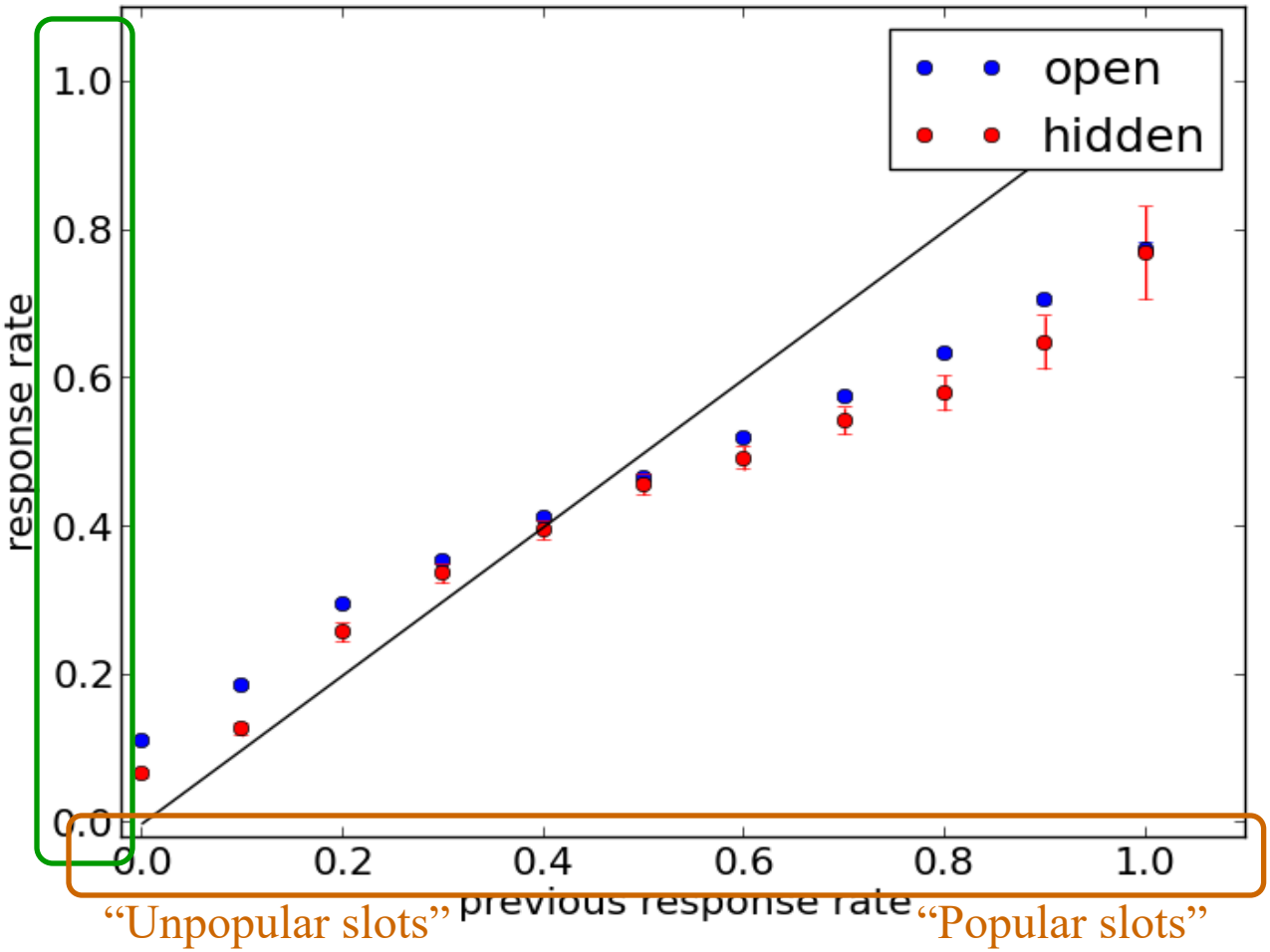
[Zou, M., Parkes, CSCW'15]

Findings for open polls:

1. More correlation with previous responses
2. Availability 35% **higher**

Based on analyzing > 340,000 real Doodle polls

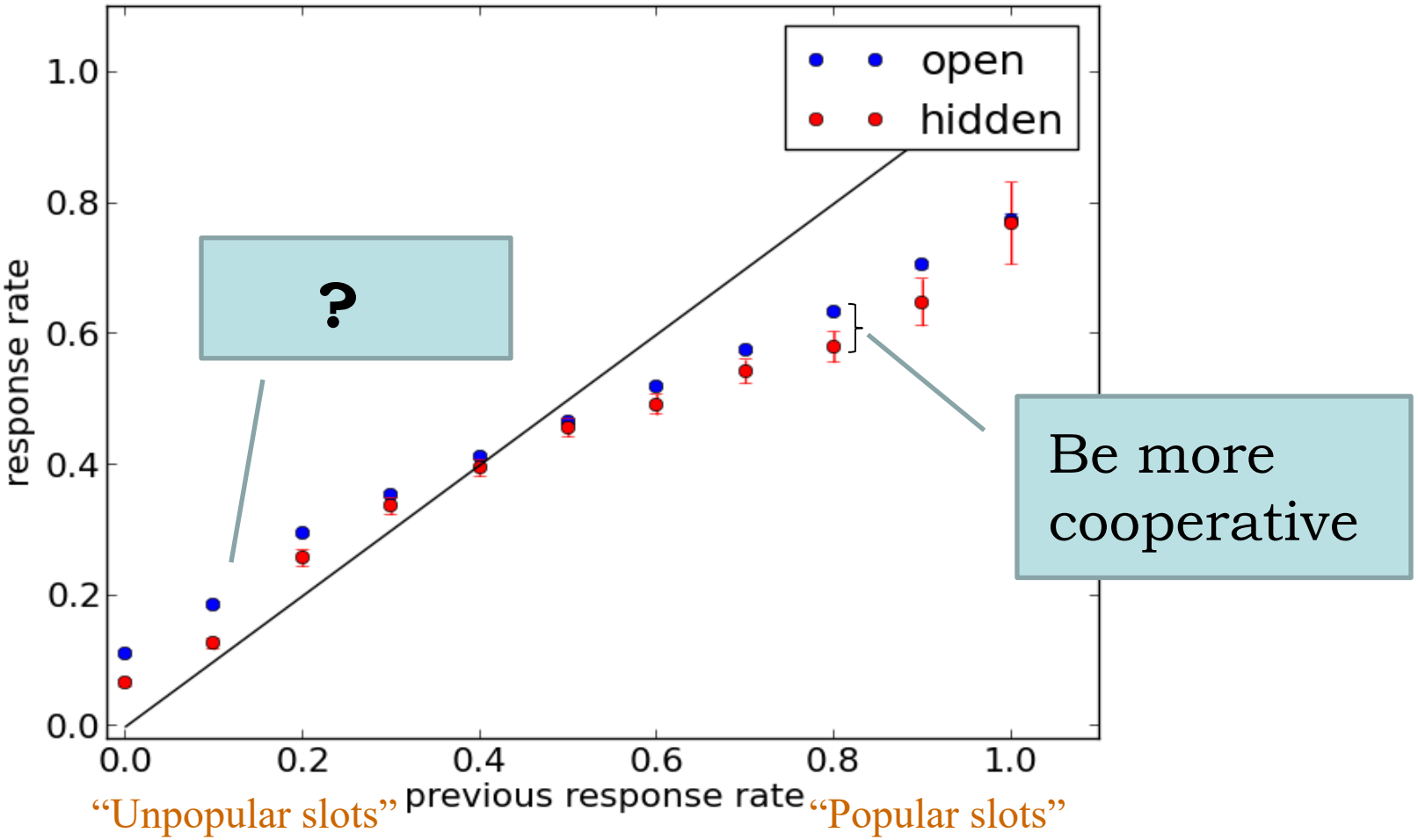
- Where are the extra available slots?



The probability that the 11th responder approves the slot

Number of previous responders who approved

- Where are the extra available slots?



Respondents strategically mark additional **unpopular** slots.

Want to *appear* cooperative!

Uncertainty in scheduling

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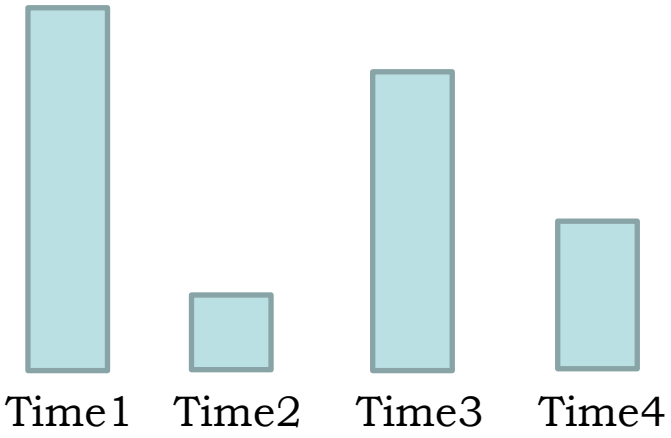

I should answer next.
I want Thu. 10am.

Uncertainty in scheduling

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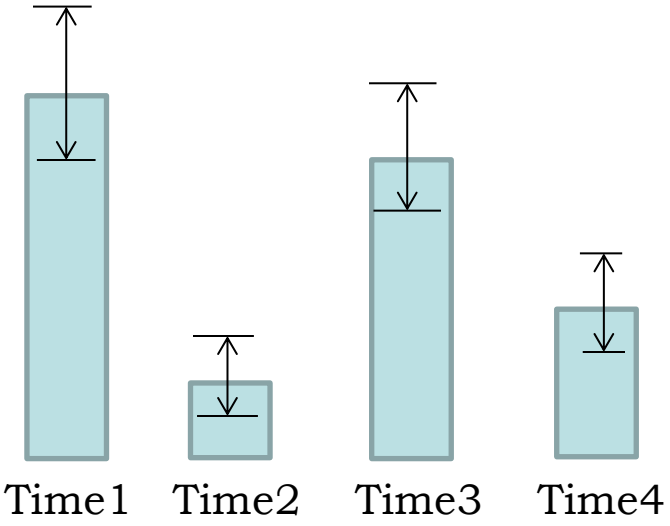
Count	6	5	3	8	6	4

Not “possible winners.”
Safe for strategic vote



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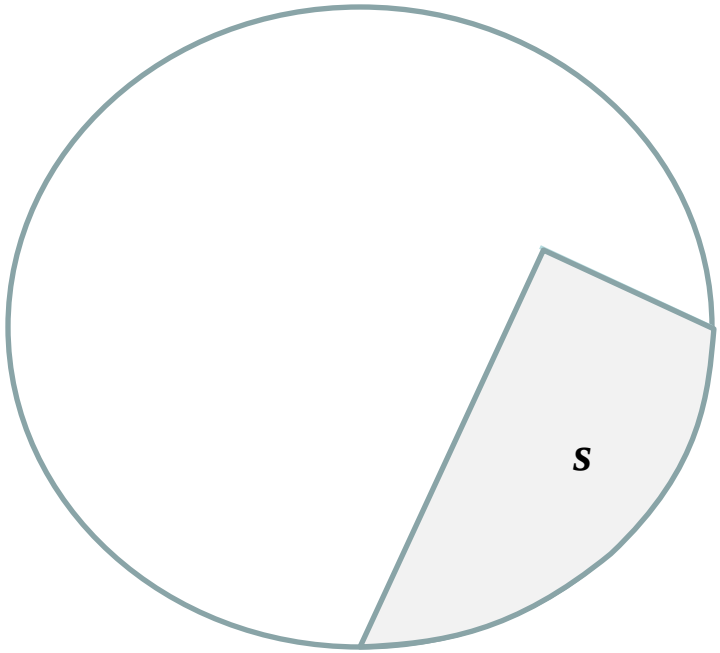
Uncertainty and modal logic



a dominates b if :

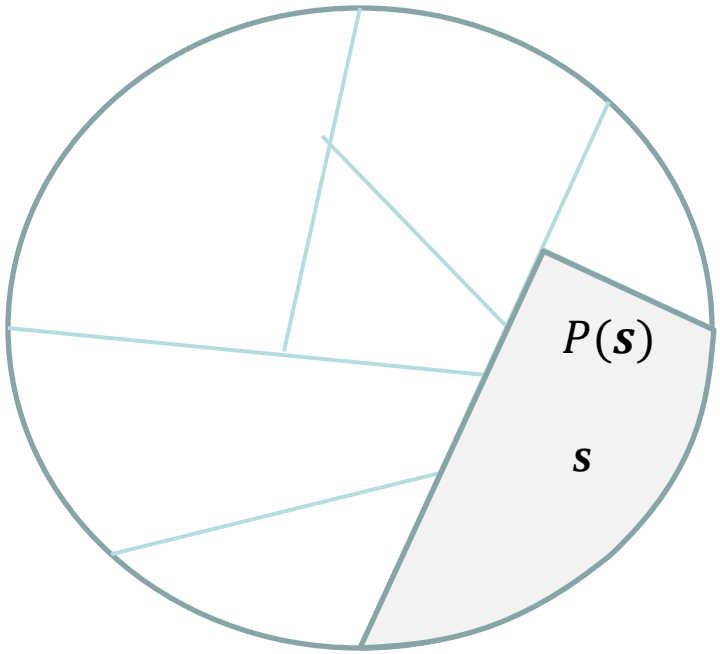
$\Box\,(f(\boldsymbol{s},a) \succcurlyeq_i f(\boldsymbol{s},b))$

$\Diamond(f(\boldsymbol{s},a) \succ_i f(\boldsymbol{s},b))$

What is the set of states accessible from $\boldsymbol{s}$?

Uncertainty and modal logic

Possible states under the S5 axioms – a partition P



"If I am in $\boldsymbol{s}$, then I know I am in $P(\boldsymbol{s})$"

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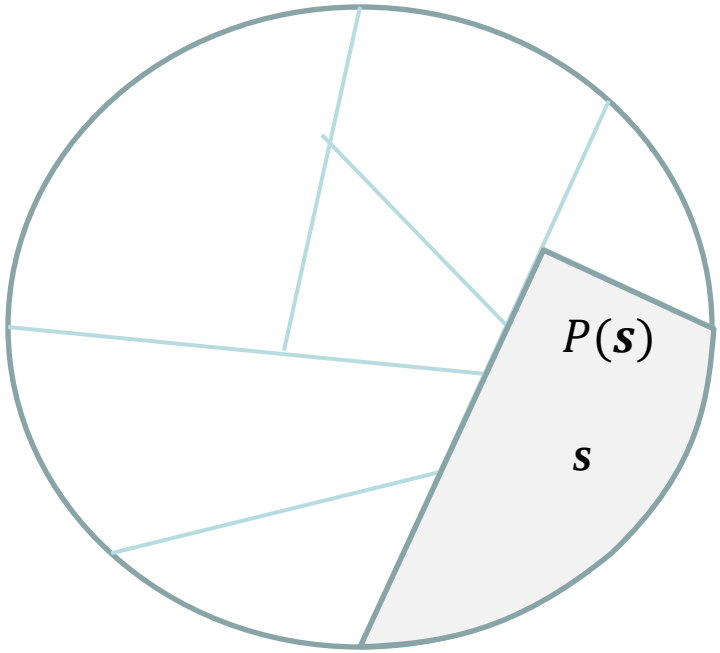

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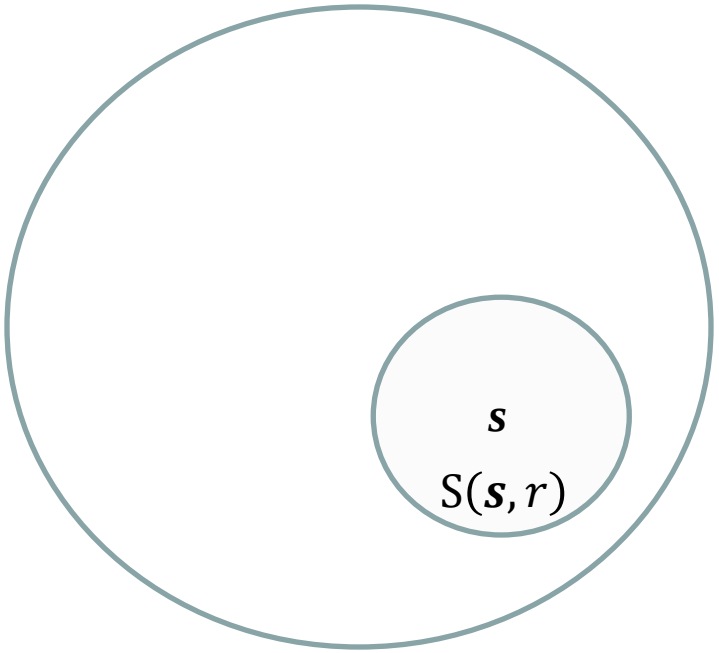
Possible states under the distance-based uncertainty



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Uncertainty and modal logic

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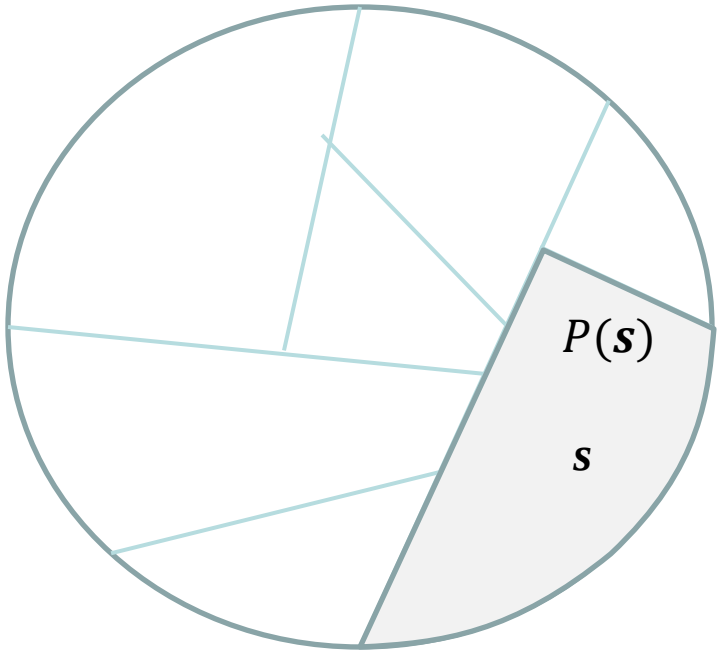

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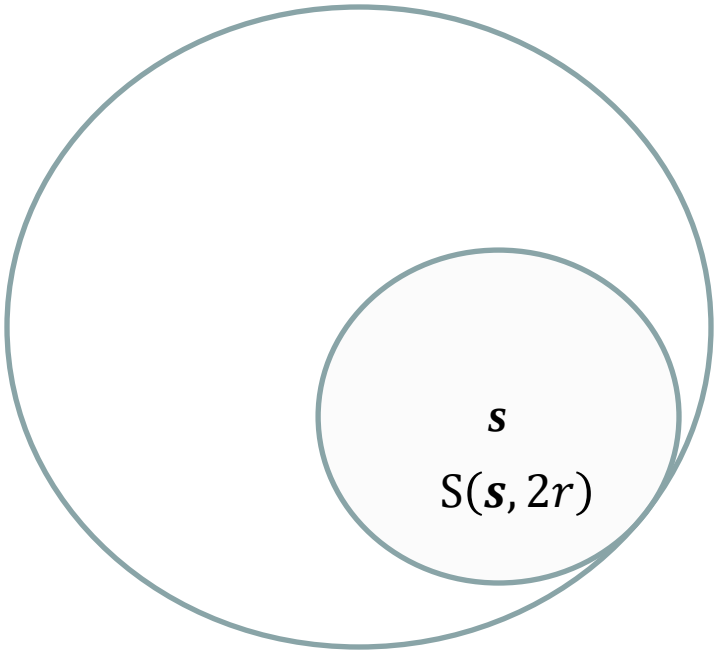

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Uncertainty and modal logic

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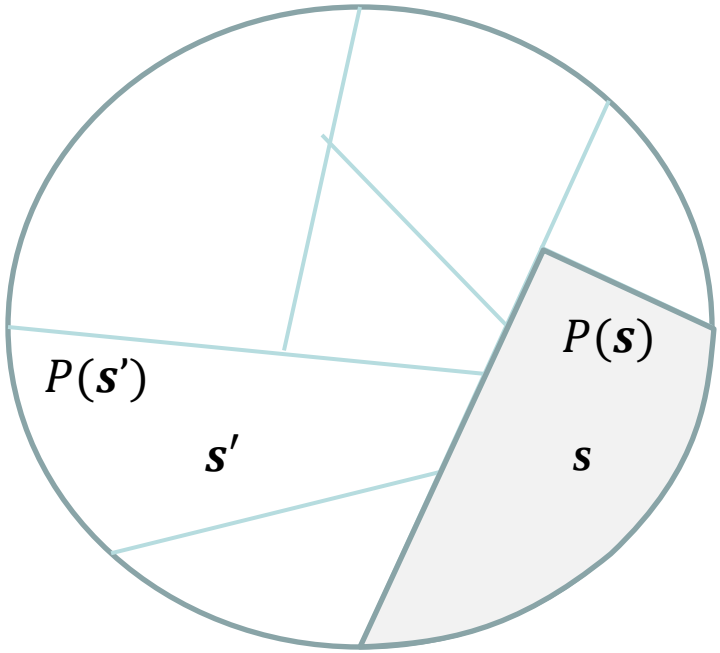

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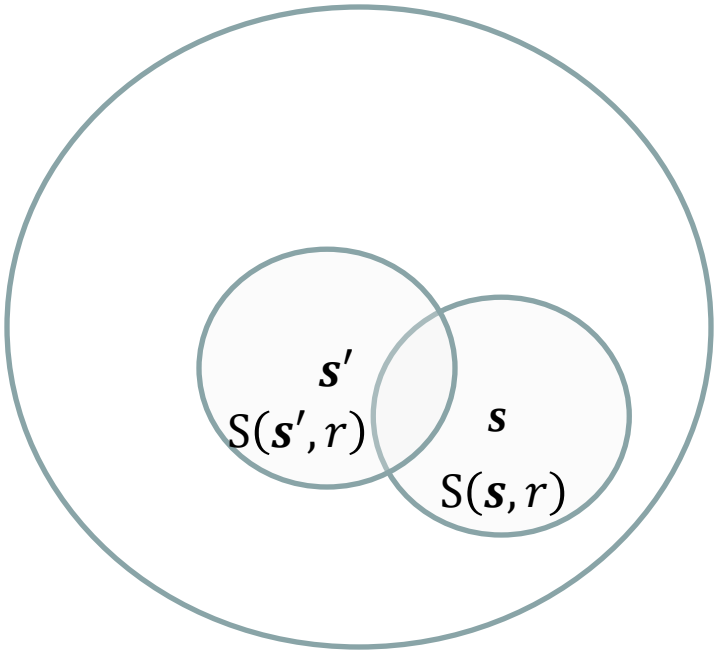


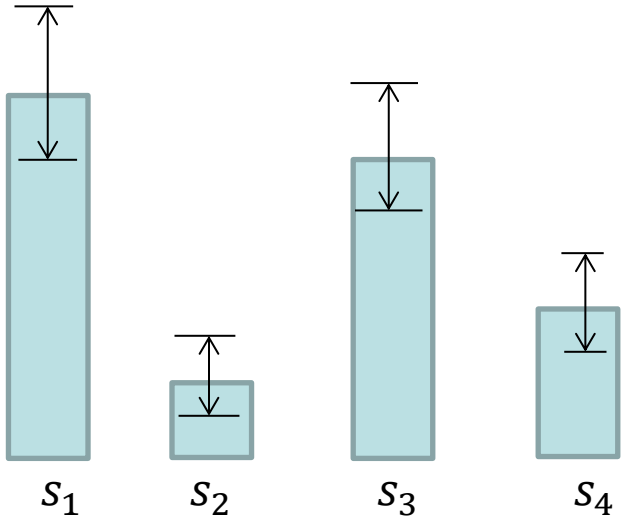
"If I am in $\boldsymbol{s}$, then I know I am close to $\boldsymbol{s}$"

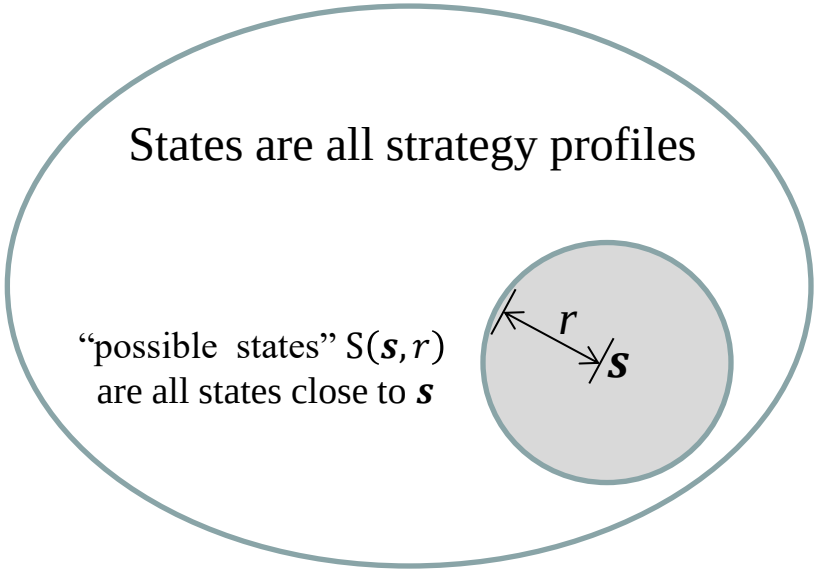
Violates transitivity

Recipe for general games

Epistemic model

$\boldsymbol{s}$ is the prospective state, induced by the current strategies





Behavioral model

- Avoid dominated actions
- Minimize worst-case cost
- Minimize worst-case regret
- Other?

Recipe for general games

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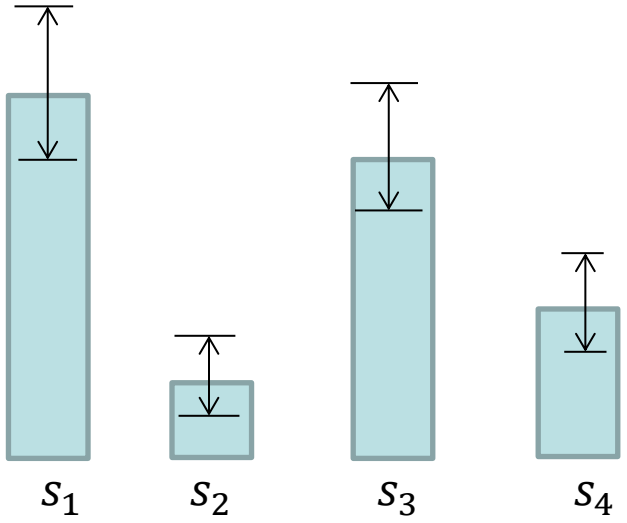
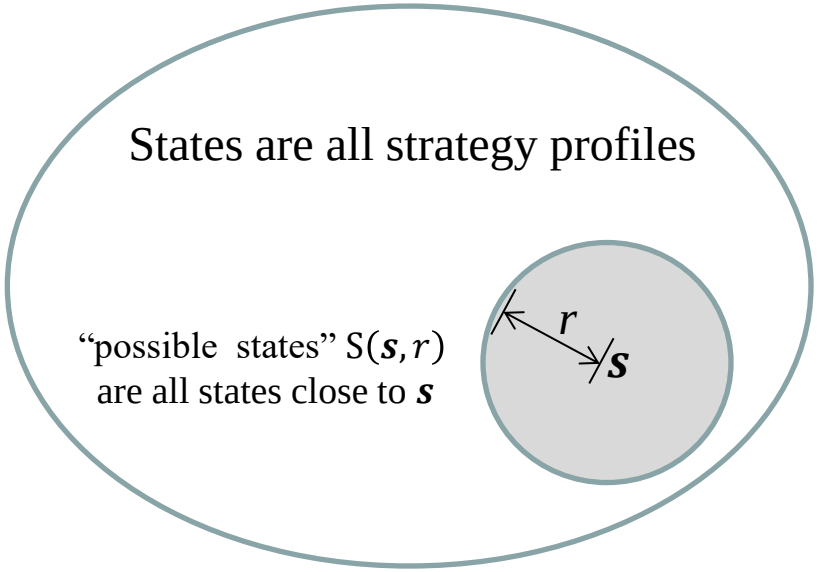




Example: Congestion Games with strict uncertainty [M. & Parkes, '15]

Behavioral model

- Avoid dominated actions
- Minimize worst-case cost
- Minimize worst-case regret
- Other?

Summary

Plurality voting	Online scheduling	?
Results	**Results**	**Results**
Behavioral model Local-dominance	**Behavioral model** Mark ``safe'' slots	**Behavioral model** **?**

Epistemic model
Distance-based uncertainty
No probabilities

Jeff Rosenschein, HUJI

James Zou, Harvard & MSR

David Parkes, Harvard

Omer Lev, HUJI

menu

The slides are based on the following papers:

- ***A Local-Dominance Theory of Voting Equilibria.*** Reshef Meir, Omer Lev, and Jeffrey S. Rosenschein. EC'14.
- ***Plurality Voting under Uncertainty****,* Reshef Meir. AAAI'15.
- ***Strategic Voting Behavior in Doodle Polls***, James Zou, Reshef Meir, and David Parkes. CSCW '15.

Other related papers:

- ***Convergence to Equilibria of Plurality Voting***, Reshef Meir, Maria Polukarov, Jeffrey S. Rosenschein and Nicholas R. Jennings. AAAI'10.
- ***A Study of Human Behavior in Voting Systems****,* Maor Tal, Reshef Meir, and Kobi Gal. AAMAS'15.
- ***Congestion Games with Distance-Based Strict Uncertainty****,* Reshef Meir and David Parkes.

Thank you!

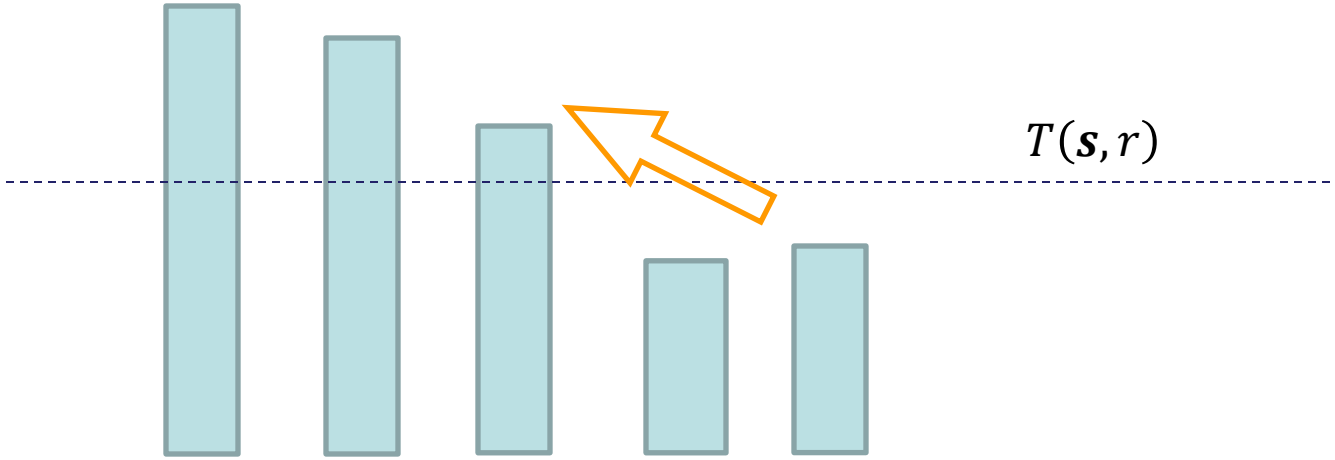
http://people.seas.harvard.edu/~rmeir/

Results

Uniform uncertainty ($r_i = r$):

Existence + Convergence if start by voting truthfully
[M., Lev, Rosenschein, EC'14]

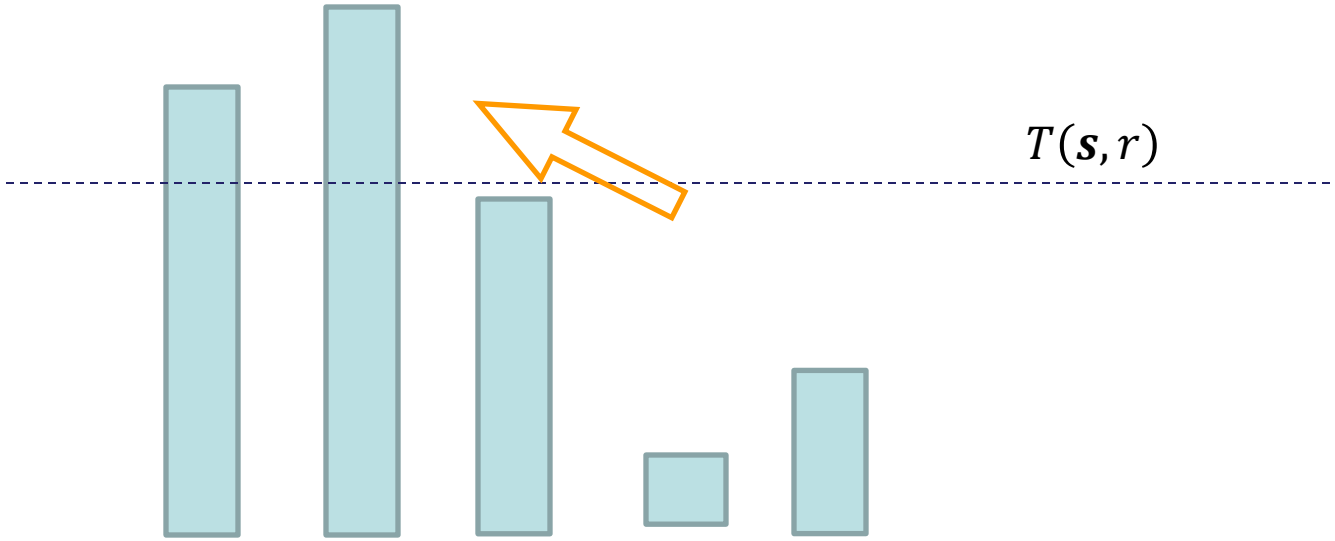
Proof intuition:



Results

Uniform uncertainty ($r_i = r$):

Existence + Convergence if start by voting truthfully
[M., Lev, Rosenschein, EC'14]



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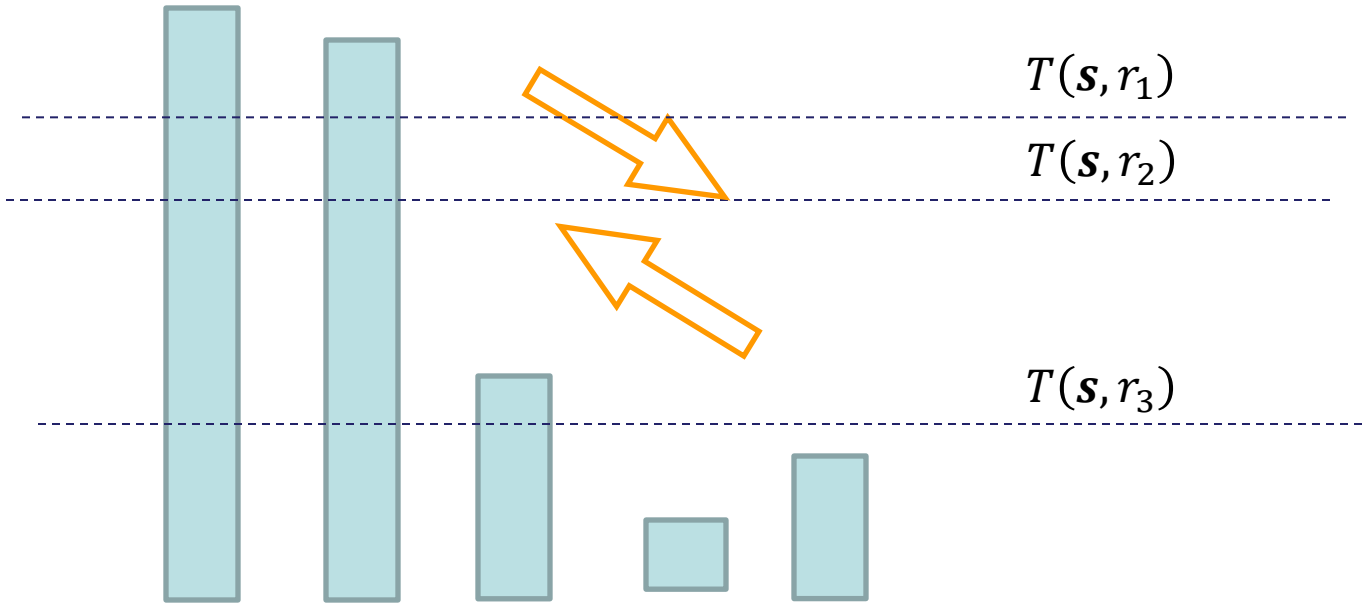
[M., Lev, Rosenschein, EC'14]



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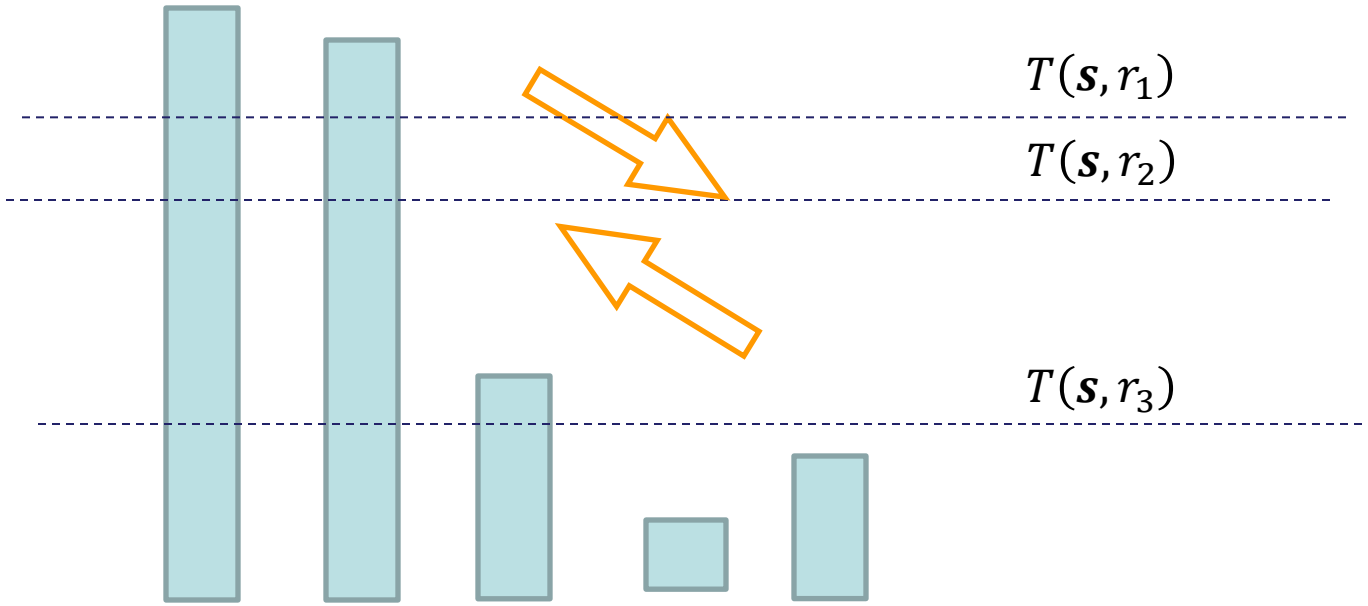
[M., Lev, Rosenschein, EC'14]



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