Proposal Mechanisms: A First Pass

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Introduction

Example: The London Interbank Offered Rate (Libor) is the interest rate at which banks can borrow from each other and plays a critical role in financial markets. Libor anchors contracts amount "to the equivalent of $45000 for every human being on the planet" MacKenzie (2008). Yet, the way this index is determined is, somewhat, a theoretical puzzle for a voting theorist.

It is determined through a highly manipulable voting rule. Indeed, the banks are asked to submit an interest rate at which their banks could borrow money. The lowest and highest quarter of the values are discarded and the Libor corresponds to the average of the remainder. In other words, the device used to determine this index is the trimmed mean rule.

Introduction

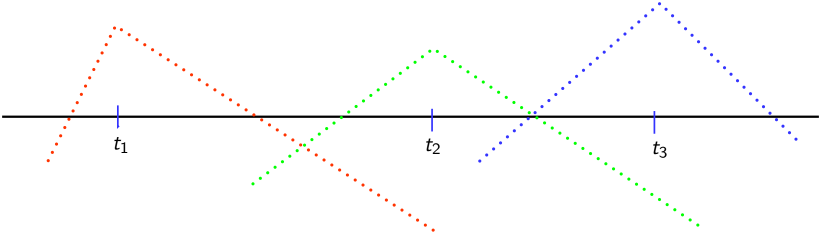
Assuming that an alternative is in the interval [0,1] and the voters are endowed with single-peaked preferences, we know that:

· 1. **Strategy-proof Rules Exist**: Strategy-proof rules were characterized by Moulin (1980)' s seminal contribution: the generalized median mechanisms.

· 2. **Do strategy-proof mechanisms really work?**: Recent strand of the literature (Sjöström et al. (2006,2007)) has submitted the properties of strategy-proof mechanisms under close strutiny. Main problem: they often exhibit a large multiplicity of equilibria. Indeed, the median rule need not lead in equilibrium to sincere behavior. Block, Nehring and Puppe (2014) confirm this prediction in an experimental setting.

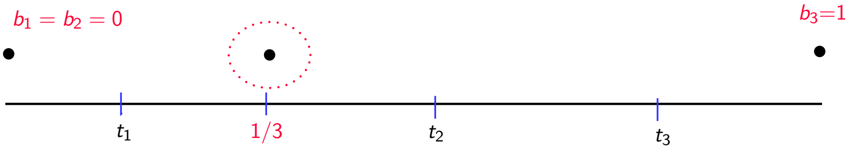
An Example

Take 3 voters with single-peaked preferences



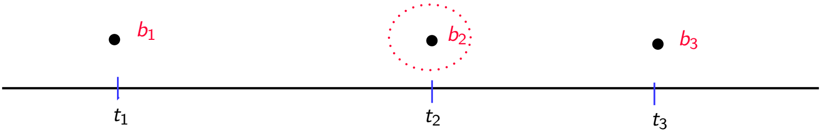
An Example: Mean Rule

In the equilibrium of the average rule, every agent adopts an extremist position 0 or 1!



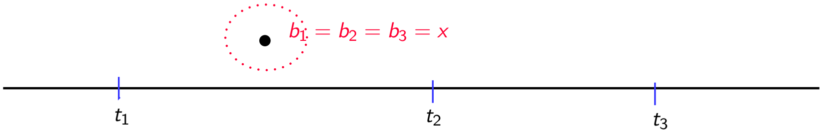
An Example: Median Mechanism

Each of them announces some point and the outcome is the median of the points.
There is an equilibrium in which every voter announces his true peak.



An Example: Median Mechanism

However, ANY alternative can be implemented in equilibrium!



Introduction

Sjöström et al. (2006,2007) suggest to focus on securely implementable mechanisms. A social choice function is securely implementable if there exists a game form that simultaneously implements it in dominant strategy equilibria and in (all) Nash equilibria.

Problem: Any securely implementable *SCC* in the single-peaked voting environment is either dictatorial or Pareto inefficient.

Question: Is there an alternative way of fixing the multiplicity of equilibria of the strategy-proof voting mechanisms ?

Introduction

This work proves that a possible manner to overcoming these problmes with strategy-proof mechanisms is by focusing on indirect mechanisms. More precisely, we design the Average Approval mechanism which exhibits the following properties:

Pure Strategy Equilibrium: The game always admits a pure strategy equilibrium in pure strategies.

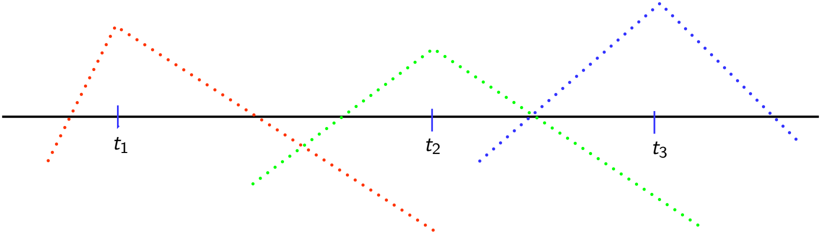
Decentralized Unanimity: The mechanism induces unanimity in the sense that there must be an equilibrium in which all players must announce a common alternative in equilibrium. Moreover, all equilibria are outcome-equivalent.

Equilibrium Outcome: The unique equilibrium outcome can be characterized as the median of the players' peaks plus some exogenous values.

Partial Revelation: There is at least one equilibrium in which all players approve of their most preferred alternative.

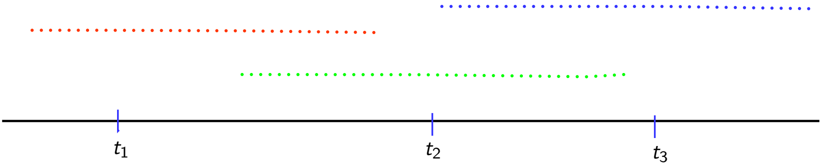
An Example

Take 3 voters with single-peaked preferences



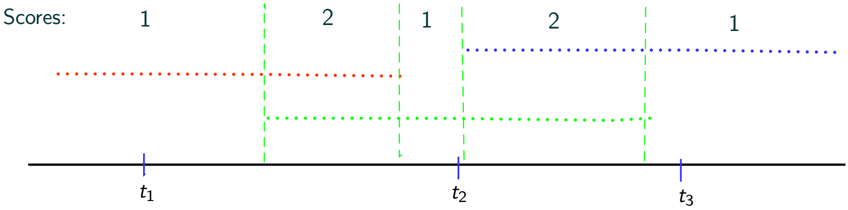
An Example: Average Approval

Each one submits a closed interval



An Example: Average Approval

These Scores generate a density function and hence an average μ_b



Introduction: A Deterministic and Continuous Mechanism

How does it work?

Each voter $i \in N$ simultaneously selects a closed interval b_i of policies (an element b_i from $\mathcal{B}$). The outcome equals *the average announced policy* in the following sense: for each profile $b = (b_1, \ldots, b_n)$, we let:

· $s_x(b) := \#\{i \in N \mid x \in b_i\}$ equals the score of alternative x given the profile b.

· $\lambda_d(b) := \sum_i \lambda_d(b_i)$ with $d = 0, 1$ denote the maximal dimension of the intervals announced in the profile b and λ_d the d-dimensional Lebesgue measure.

Introduction: A Deterministic and Continuous Mechanism

How does it work?

· $f_b(x) = \frac{s_x(b)}{\lambda_d(b)}$ for each $x \in [0,1]$. f_b is a well-defined density function for any profile b.

· $\mu_b := \int_{[0,1]} x f_b(x) dx$ denotes the average outcome with $\mu_b \in [0,1]$.

The Average Approval mechanism implements μ_b as the bargaining outcome so that $u_i(b) = u_i(\mu_b)$ for any $i \in N$ and any profile b.

Preferences are single-peaked and we let t_i denote voter i's peak. When x is the implemented policy, the utility for player i equals $u_i(x)$ with $u_i(x') < u_i(x'')$ when $x' < x'' \leq t_i$ and when $t_i \leq x'' < x'$.

Properties: Best Responses

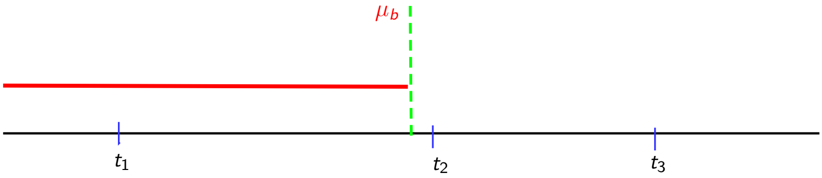
For each proposal profile b, we let $L(b)$ and $R(b)$ denote the set of alternatives located respectively to the left and to the right of μ_b so that $L(b) = \{x \in [0, 1] \mid x \leq \mu_b\}$ and $R(b) = \{x \in [0, 1] \mid x \leq \mu_b\}$.

Lemma: Let b denote a proposal profile. If b_i is a best response to b_{-i}, then

$$b_i = \begin{cases} L(b) & \text{if} \quad t_i < \mu_b, \\ R(b) & \text{if} \quad t_i > \mu_b. \end{cases}$$

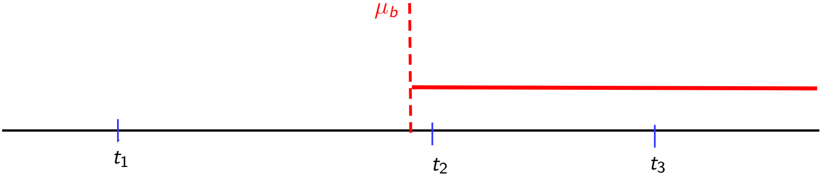
An Example: Best Responses

If $t_i < \mu_b$, the a unique best response: $[0, \mu_b]$



An Example: Best Responses

If $t_i > \mu_b$, then the unique best response: $[\mu_b, 1]$



Properties: Description of Equilibria

Theorem 1: The Average Approval mechanism admits an equilibrium in Pure Strategies for any distribution of the voters' peaks.

Theorem 2: All equilibria b^* implement the same alternative μ_b^*. Among them, there is at least one in which all voters include μ_b^* in their interval.

Equilibrium Outcome as a Generalized Median

For any finite collection of points $x_1, \ldots, x_m$ in $[0, 1]$, we let $m(x_1, \ldots, x_m)$ denote their median, that is the smallest number $m(x_1, \ldots, x_m) \in x_1, \ldots, x_m$, which satisfies:

$$\frac{1}{m}\#\{x_i \mid x_i \leq m(x_1, \ldots, x_m)\} \geq \frac{1}{2} \text{ and } \frac{1}{m}\#\{x_i \mid x_i \geq m(x_1, \ldots, x_m)\} \geq \frac{1}{2}.$$

If m is odd, the median is unique while if it is even, there are two such numbers, in which case we denote the smallest of them as the median.

Two Players

Let $n = 2$ with $t_1 \leq t_2$ denoting their respective peaks. In the unique equilibrium, the alternative selected is $m(t_1, t_2, 1/2)$. To obtain this outcome, the equilibrium proposal $b^* = (b_1^*, b_2^*)$ satisfies:

$$b_1^* = \begin{cases} [0, t_2], \\ [0, 1/2], \\ [2t_1 - 1, t_1] \end{cases} \quad b_2^* = \begin{cases} [t_2, 2t_2] & \text{if} \quad m(t_1, t_2, 1/2) = t_2, \\ [1/2, 1] & \text{if} \quad m(t_1, t_2, 1/2) = 1/2, \\ [t_1, 1] & \text{if} \quad m(t_1, t_2, 1/2) = t_1. \end{cases}$$

In each equilibrium, both players include the implemented policy in their proposal. For instance, take the case with $t_1 = 1/4 < t_2 = 1/3 < 1/2$. The equilibrium outcome equals $1/3$ and the proposal profile b^* equals $([0, 1/3], [1/3, 2/3])$ with $\mu_b^* = 1/3$. In this equilibrium, Player 1 cannot do better than approving all the alternatives to the left of $1/3$ and Player 2 obtains his peak and hence has no profitable deviation.

Equilibrium Outcome as a Generalized Median

For each $j = 1, \ldots, n-1$, we let b^j denote the proposal profile with $n-j$ players playing $[0, \mu_b^j]$ and j voters selecting the interval $[\mu_b^j, 1]$. We let $\kappa_j \equiv \mu_{b^j}$ so that:

$$\kappa_j = \frac{\sqrt{j}}{\sqrt{n-j} + \sqrt{j}} \text{ and } \kappa_1 \leq \kappa_2 \leq \ldots \leq \kappa_{n-1}.$$

Theorem 3: The alternative $e(t_1, t_2, \ldots, t_n)$ implemented by the AA mechanism in equilibrium equals:

$$e(t_1, t_2, \ldots, t_n) = m(t_1, t_2, \ldots, t_n, \kappa_1, \ldots, \kappa_{n-1})$$

.

Conclusion

We propose an indirect mechanism: the Average Approval one. This deterministic and continuous mechanism exhibits interesting properties:

1. It is not Strategy-Proof (since this condition is vacuous in our setting) but it is partially revealing.

2. It leads to Consensual Decisions in every equilibrium.

3. It admits an equilibrium in pure strategies and the equilibrium outcome is unique.

4. As the Revelation principle anticipates, this outcome can be represented by a direct mechanism. In this case, it coincides with the generalized median of the peaks of the voters.

Conclusion

In other words, this paper suggests that “proposal mechanims” where agents’ strategies are a subset of the outcome space are a promising research venue.

Indeed, Proposal mechanisms can exhibit appealing features than direct mechanisms simply cannot by their nature. As we have shown, one can obtain an agreement on an outcome AND partially revealing strategies in a pure strategy equilibrium.

What can be achieved with set-based mechanisms is left for future research.

Conclusion

Our results do not clash with the Revelation Principle. As Myerson (2008) argues, this principle states that indirect mechanisms can be simulated by an equivalent incentive-compatible direct-revelation mechanism. However, we are not anymore concerned with ONE equilibrium but with the entire set.

Moreover, an alternative way of stating our result is to say that Indirect Mechanisms are concerned with "HOW DO WE IMPLEMENT a SCF? whereas the Direct Mechanisms deal with "WHICH SCFs CAN WE IMPLEMENT?", as the Revelation Principle shows. It seems natural/intuitive that players prefer to reach a unanimous agreement rather than one imposed by a third party. Still, this is not present in the payoff functions of the players.