Justified Representation in Approval-Based Committee Voting

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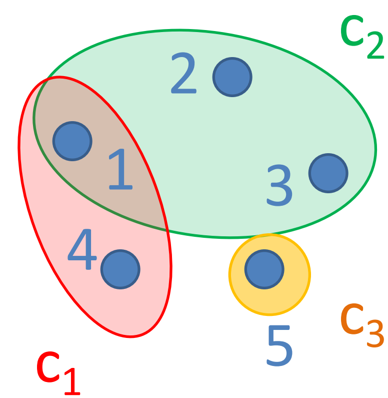
Voting with Approval Ballots

- A set of candidates C
- n voters $\{1, \ldots, n\}$
- Each voter i **approves** a subset of candidates $A_i \subseteq C$
- <u>Goal</u>: select k winners (a **committee**)



1: c_1, c_2
2: c_2
3: c_2
4: c_1
5: c_3

Outline

- Approval-based multiwinner rules
- Justified Representation (JR)
- Which rules satisfy JR?
- Extended Justified Representation (EJR)
- (E)JR and core stability

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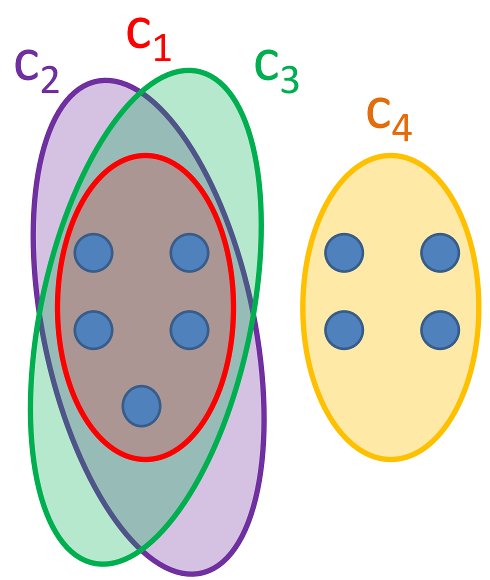
Approval Voting (AV)

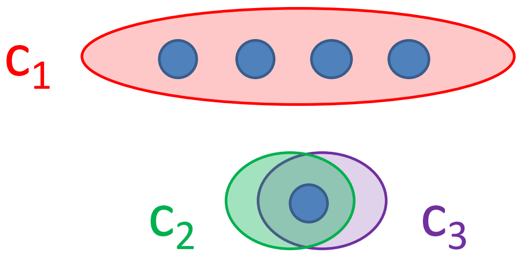
- Each candidate gets one point from each voter who approves her
- k candidates with the highest score are selected
  - ties broken deterministically



for $k=3$
AV outputs
$\{c_1, c_2, c_3\}$

Minimax Approval Voting (MAV)

- Brams, Kilgour & Sanver '07
- Distance from ballot A_i to a committee W: $d(A_i, W) = |A_i \setminus W| + |W \setminus A_i|$
- Goal: select a size-k committee that minimizes $\max_i d(A_i, W)$



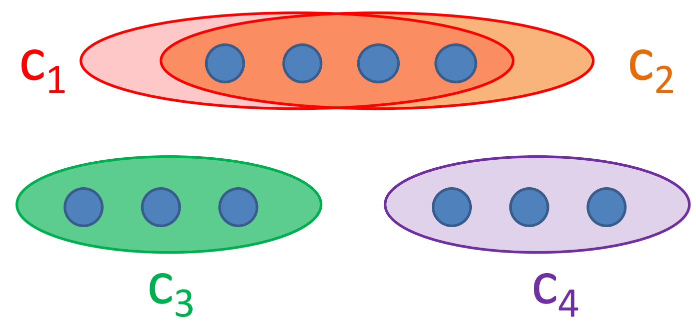
for $k=1$

AV outputs c_1,

MAV outputs c_2 or c_3

Satisfaction Approval Voting (SAV)

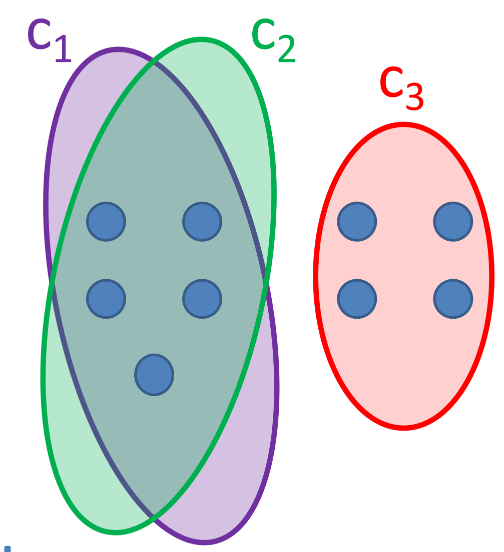
- Brams & Kilgour ’14
- Voter i scores committee W as $|A_i \cap W| / |A_i|$
- <u>Goal</u>: select a size-k committee with the maximum score



for k=2
AV outputs $\{c_1, c_2\}$,
SAV outputs $\{c_3, c_4\}$

Proportional Approval Voting (PAV)

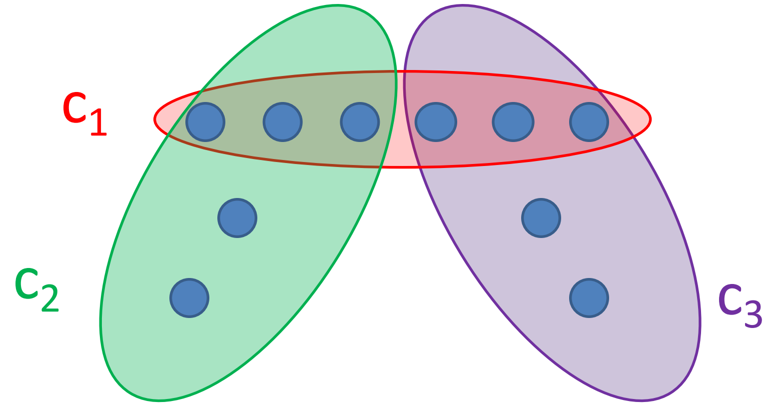
- Simmons '01
- Voter i derives utility of 1 from her 1^{st} approved candidate, $1/2$ from 2^{nd}, $1/3$ from 3^{rd}, etc.
- $u_i(W) = 1 + 1/2 + \ldots + 1/|W \cap A_i|$
- <u>Goal</u>: select a size-k committee W that maximizes $u(W) = \Sigma_i u_i(W)$



for $k=2$
AV outputs $\{c_1, c_2\}$,
PAV outputs $\{c_1, c_3\}$ or $\{c_2, c_3\}$

Reweighted Approval Voting (RAV)

- Thiele, early 20th century
- Sequential version of PAV
- Initialize:
  $\omega(i) = 1$ for all i, $W = \varnothing$
- Repeat k times:
  – add to W a candidate with max approval weight
    $\omega(c) = \Sigma_{\text{i approves c}}\, \omega(i)$
  – update the weight of each voter to $\omega(i) = 1/(1+|A_i \cap W|)$



for $k=2$
PAV outputs $\{c_2, c_3\}$,
RAV outputs
$\{c_1, c_2\}$ or $\{c_1, c_3\}$

Generalizing PAV and RAV: Arbitrary Weights

- PAV and RAV both use weight vector $(1, 1/2, 1/3, \ldots)$
- We can use an arbitrary weight vector $(w_1, w_2, \ldots)$ with $w_1 = 1$, $w_1 \geq w_2 \geq \ldots$ instead: $(w_1, w_2, \ldots)$-PAV and $(w_1, w_2, \ldots)$-RAV
- $(1, 0, \ldots)$-RAV: choose candidates one by one to cover as many uncovered voters as possible at each step (Greedy Approval Voting (GAV))

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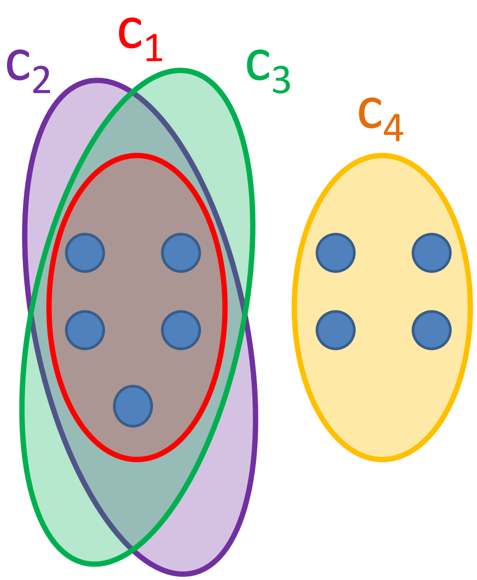
Representation

- 5 voters get 3 representatives,
  4 voters get 0 representatives

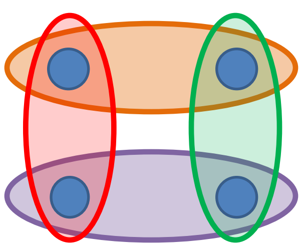
- Intuition: each cohesive group of voters of size n/k “deserves” at least one representative



for $k=3$
AV outputs
$\{c_1, c_2, c_3\}$

First Attempt: Strong Justified Representation

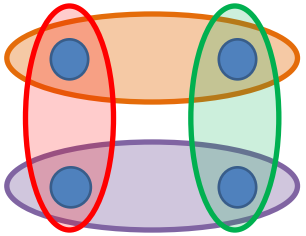
- Definition: a committee W provides strong justified representation (SJR) for a list of ballots $(A_1,\ldots, A_n)$ and committee size k if for every set of voters X with $|X| \geq n/k$ and $\cap_{i \in X} A_i \neq \varnothing$ it holds that W contains at least one candidate from $\cap_{i \in X} A_i$.
- Bad news: for some profiles, no committee provides SJR



k=2

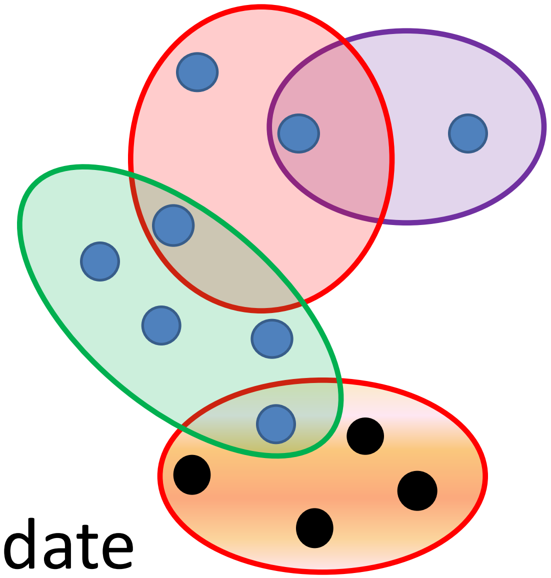
Justified Representation

- Definition: a committee W provides justified representation (JR) for a list of ballots $(A_1,\ldots, A_n)$ and committee size k if for every set of voters X with $|X| \geq n/k$ and $\cap_{i \in X} A_i \neq \varnothing$ it holds that W contains at least one candidate from $\cup_{i \in X} A_i$.
  - Equivalently: there does not exist a cohesive group of n/k voters that is totally unrepresented



Can We Always Satisfy JR?

- Claim: GAV (aka (1, 0, …)-RAV) always outputs a committee that provides JR.
- Proof:
  - Suppose after k steps we have n/k uncovered voters who all approve a
  - a's weight is ≥ n/k
  - then at each step we chose a candidate that covered ≥ n/k uncovered voters
  - thus we should have covered all n voters



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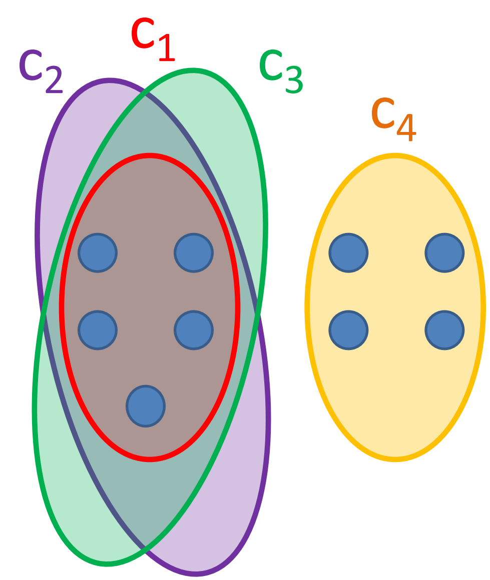
Rules that fail JR

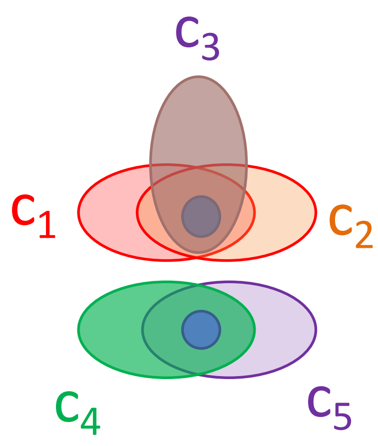
- AV fails JR for $k \geq 3$
- SAV fails JR for $k \geq 2$
- MAV fails JR for $k \geq 2$
  - except if each ballot is of size k and ties are broken in favour of JR



for $k=3$
AV outputs
$\{c_1, c_2, c_3\}$

SAV Fails JR

- SAV:
  - voter i scores committee W as $|A_i \cap W|/|A_i|$
  - SAV select a size-k committee with the maximum score
- SAV fails JR



$k=n=2$

SAV outputs $\{c_4, c_5\}$

PAV, RAV and JR

- Theorem: PAV satisfies JR
  - $(w_1, w_2, \ldots)$-PAV satisfies JR iff $w_j \leq 1/j$ for all j

- Theorem: RAV fails JR for $k \geq 10$
  - $k = 3, \ldots, 9$ is open!
  - $(w_1, w_2, \ldots)$-RAV fails JR if $w_2 > 0$
  - $(1, 0, \ldots)$-RAV is GAV and satisfies JR
  - $(1, 1/n, \ldots)$-RAV satisfies JR

PAV Satisfies JR

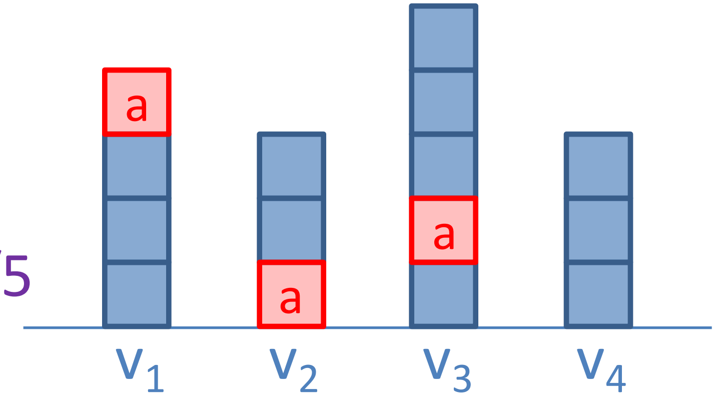
- $u_i(W) = 1 + 1/2 + \ldots + 1/|W \cap A_i|$
- Goal: select a size-k committee W that maximizes $u(W) = \Sigma_i u_i(W)$
- Theorem: PAV satisfies JR
- Proof idea:
  - if not, there is some $c \in C$ that could increase the total utility by $\geq n/k$
  - we will show that some candidate $a \in W$ contributes $< n/k$

PAV Satisfies JR

- Proof:
  - MC(a) := u(W) - u(W \ a): marginal utility of a
  - MC(a, i) : = u_i (W) - u_i (W \ a): marginal utility of a for i
  - Σ_a MC(a) =
  - MC(a) < n/k for some a in W
  - u(W ∪ c \ a) > u(W)

MC(a, 1) = 1/4
MC(a, 2) = 1/3
MC(a, 3) = 1/5

MC(a) = 1/4+1/3+1/5



Summary: JR

	Satisfies JR	
AV	No	
SAV	No	
MAV	No	
PAV	Yes	
RAV	No	
GAV	Yes	

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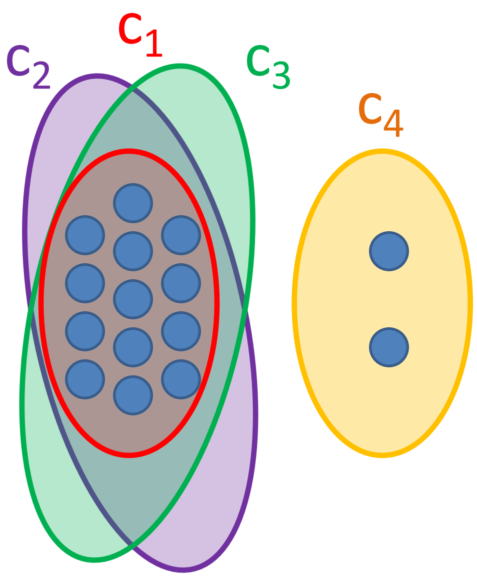
Is JR Enough?

- Should we choose c_4 ???
- Perhaps a **very large** cohesive group of voters "deserves" several representatives?
- <u>Idea</u>: if n/k voters who agree on a candidate "deserve" one representative, then maybe $\ell \cdot n/k$ voters who agree on ℓ candidates "deserve" ℓ representatives?



Extended Justified Representation

- Definition: a committee W provides extended justified representation (EJR) for a list of ballots $(A_1,\dots, A_n)$ and committee size k if for every $\ell > 0$, every set of voters X with $|X| \geq \ell \cdot n/k$ and $|\bigcap_{i\in X} A_i| \geq \ell$ it holds that $|W \cap A_i| \geq \ell$ for at least one $i \in X$.

- $\ell = 1$: justified representation

Satisfying EJR

- Observation: GAV fails EJR
- Theorem: PAV satisfies EJR
  - $(w_1, w_2, \ldots)$-PAV fails EJR if $(w_1, w_2, \ldots) \neq (1, 1/2, 1/3, \ldots)$
- But PAV is NP-hard to compute [AGGMMW'14]
  - Are there any other rules satisfying EJR?
- Theorem: checking if a committee provides EJR is coNP-complete
- Open: complexity of finding an EJR committee

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A Cooperative Game

- Given k and $(A_1, \ldots, A_n)$, consider NTU game with players $\{1, \ldots, n\}$
  - each coalition of size x with $\ell \cdot n/k \le x \le (\ell+1) \cdot n/k$ can "purchase" ℓ alternatives
  - players evaluate committees using PAV utility function
  - a coalition has a profitable deviation if they can purchase a set of candidates that is strictly preferred by everybody in the coalition
  - core: outcomes w/o profitable deviations

(E)JR and Core Stability

- <u>Theorem:</u> Committee provides JR iff no coalition of size $\leq \lceil n/k \rceil$ has a profitable deviation.
- <u>Theorem:</u> Committee provides EJR iff for every $\ell \geq 0$, no coalition X with $\ell \cdot n/k \leq |X| \leq (\ell+1) \cdot n/k$ and $|\cap_{i \in X} A_i| \geq \ell$ has a profitable deviation.
  - not true for arbitrary coalitional deviations!
- Open problems:
  - Is the core always non-empty?
  - Find a rule that selects from the core (if non-empty)

Conclusion

- New properties for approval-based committee voting rules
  - capture representation
  - EJR characterizes PAV weight vector (1, ½, …)
- Open problems:
  - tractable rules satisfying EJR
  - core-selecting rules
  - restricted domains

	JR	EJR
AV	No	No
SAV	No	No
MAV	No	No
PAV	Yes	Yes
RAV	No	No
GAV	Yes	No

Thank you!